

California Tutorials are designed specifically for the California Common Core State Standards and the California Next Generation Science Standards to prepare students for the Smarter Balanced Assessment Consortium (SBAC) exams and the California Science Tests.

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

## 1. EXPRESSIONS, EQUATIONS, AND INEQUALITIES

### ● FORMULATING AND SIMPLIFYING ALGEBRAIC EXPRESSIONS

- **A-SSE.1.a** Interpret parts of an expression, such as terms, factors, and coefficients.
- **A-SSE.2** Use the structure of an expression to identify ways to rewrite it.

### ● FORMULATING AND SOLVING EQUATIONS FROM WORD PROBLEMS

- **A-CED.1** Create equations and inequalities in one variable including ones with absolute value and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

### ● FORMULATING AND SOLVING INEQUALITIES FROM WORD PROBLEMS

- **A-CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- **A-CED.1** Create equations and inequalities in one variable including ones with absolute value and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

### ● LITERAL EQUATIONS

- **A-CED.1** Create equations and inequalities in one variable including ones with absolute value and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
- **A-CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

### ● SUMS OF GEOMETRIC SEQUENCES

- **A-SSE.4** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

## 2. EXPONENTIAL AND LOGARITHMIC FUNCTIONS

### ● LOGARITHMIC FUNCTIONS

- **F-IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of

the function.

- **F-BF.4.a** Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse.
- **F-LE.4** For exponential models, express as a logarithm the solution to  $ab$  to the  $ct$  power  $= d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.
- **F-IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **F-IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- **F-IF.7.e** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

#### ● EVALUATING LOGARITHMIC EXPRESSIONS

- **A-SSE.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **A-SSE.2** Use the structure of an expression to identify ways to rewrite it.
- **F-LE.4.3** Understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values.
- **F-LE.4** For exponential models, express as a logarithm the solution to  $ab$  to the  $ct$  power  $= d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.
- **F-LE.4.2** Use the definition of logarithms to translate between logarithms in any base.

#### ● SOLVING EXPONENTIAL EQUATIONS

- **F-IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- **F-BF.4.a** Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse.
- **F-IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **F-IF.7.e** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- **F-LE.4** For exponential models, express as a logarithm the solution to  $ab$  to the  $ct$  power  $= d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.

#### ● SOLVING LOGARITHMIC EQUATIONS

- **F-BF.4.a** Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse.
- **F-LE.4.2** Use the definition of logarithms to translate between logarithms in any base.
- **F-LE.4.3** Understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values.
- **F-IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

### 3. POLYNOMIALS

#### ● POLYNOMIAL BASICS

- **A-SSE.1.a** Interpret parts of an expression, such as terms, factors, and coefficients.
- **A-APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

#### ● ADDITION AND SUBTRACTION OF POLYNOMIALS

- **A-SSE.2** Use the structure of an expression to identify ways to rewrite it.
- **A-APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

#### ● MULTIPLICATION OF POLYNOMIALS

- **A-SSE.2** Use the structure of an expression to identify ways to rewrite it.
- **A-APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

#### ● **DIVISION OF POLYNOMIALS**

- **A-APR.2** Know and apply the Remainder Theorem: For a polynomial  $p(x)$  and a number  $a$ , the remainder on division by  $x - a$  is  $p(a)$ , so  $p(a) = 0$  if and only if  $(x - a)$  is a factor of  $p(x)$ .
- **A-APR.6** Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.
- **A-SSE.2** Use the structure of an expression to identify ways to rewrite it.

### 4. FACTORING POLYNOMIALS AND THE FACTOR THEOREM

#### ● **FACTORING CUBIC POLYNOMIALS**

- **A-SSE.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **A-SSE.2** Use the structure of an expression to identify ways to rewrite it.
- **A-APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- **A-APR.4** Prove polynomial identities and use them to describe numerical relationships.

#### ● **FACTORING HIGHER ORDER POLYNOMIALS**

- **A-SSE.1.a** Interpret parts of an expression, such as terms, factors, and coefficients.
- **A-SSE.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **A-SSE.2** Use the structure of an expression to identify ways to rewrite it.
- **A-APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- **A-APR.4** Prove polynomial identities and use them to describe numerical relationships.

#### ● **FACTOR THEOREM AND REMAINDER THEOREM**

- **A-APR.6** Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.
- **A-APR.2** Know and apply the Remainder Theorem: For a polynomial  $p(x)$  and a number  $a$ , the remainder on division by  $x - a$  is  $p(a)$ , so  $p(a) = 0$  if and only if  $(x - a)$  is a factor of  $p(x)$ .

### 5. POLYNOMIALS AND POLYNOMIAL IDENTITIES

#### ● **GRAPHS OF POLYNOMIAL FUNCTIONS**

- **A-APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- **F-IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **F-IF.7.c** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- **F-BF.3** Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $(k)$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

#### ● **REPRESENTATIONS OF QUADRATIC FUNCTIONS**

- **A-SSE.2** Use the structure of an expression to identify ways to rewrite it.
- **F-IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- **A-CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on

coordinate axes with labels and scales.

- **A-CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- **A-APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- **F-IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **F-IF.7.c** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- **A-CED.1** Create equations and inequalities in one variable including ones with absolute value and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
- **F-BF.3** Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

#### ● POLYNOMIAL IDENTITIES

- **A-SSE.2** Use the structure of an expression to identify ways to rewrite it.
- **A-APR.4** Prove polynomial identities and use them to describe numerical relationships.
- **A-APR.5** Know and apply the Binomial Theorem for the expansion of  $(x + y)^n$  in powers of  $x$  and  $y$  for a positive integer  $n$ , where  $x$  and  $y$  are any numbers, with coefficients determined for example by Pascal's Triangle.

#### ● POLYNOMIAL IDENTITIES AND COMPLEX NUMBERS

- **N-CN.8** Extend polynomial identities to the complex numbers.
- **A-SSE.2** Use the structure of an expression to identify ways to rewrite it.
- **A-APR.4** Prove polynomial identities and use them to describe numerical relationships.
- **N-CN.9** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

## 6. RADICAL EQUATIONS AND FUNCTIONS

#### ● ANALYZING GRAPHS OF SQUARE ROOT FUNCTIONS

- **F-BF.3** Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- **F-IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **F-IF.7.b** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- **F-BF.4.a** Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse.
- **F-IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

#### ● SOLVING SQUARE ROOT EQUATIONS

- **A-REI.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

## 7. RATIONAL EXPRESSIONS, EQUATIONS, AND FUNCTIONS

#### ● OPERATIONS WITH RATIONAL EXPRESSIONS

- **A-APR.7** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
- **A-SSE.2** Use the structure of an expression to identify ways to rewrite it.

#### ● ANALYZING GRAPHS OF RATIONAL FUNCTIONS

- **F-BF.3** Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- **F-IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **F-IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

### ● SOLVING RATIONAL EQUATIONS

- **A-REI.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- **A-CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

### ● MODELING SITUATIONS WITH RATIONAL FUNCTIONS

- **A-SSE.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **A-CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **A-CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- **A-REI.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- **A-CED.1** Create equations and inequalities in one variable including ones with absolute value and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

## 8. TRIGONOMETRY

### ● RADIANS AND THE UNIT CIRCLE

- **F-TF.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- **F-TF.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

### ● TRIGONOMETRIC FUNCTIONS

- **F-IF.7.e** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- **F-TF.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- **F-TF.5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
- **F-TF.2.1** Graph all 6 basic trigonometric functions.

### ● LAWS OF SINE AND COSINE

- **G-SRT.9** Derive the formula  $A = \frac{1}{2} ab \sin(C)$  for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
- **G-SRT.10** Prove the Laws of Sines and Cosines and use them to solve problems.
- **G-SRT.11** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

## 9. CONIC SECTIONS

### ● COMPLETING THE SQUARE

- **G-GPE.3.1** Given a quadratic equation of the form  $ax^2 + by^2 + cx + dy + e = 0$ , use the method for completing the square to put the equation into standard form; identify whether the graph of the equation is a circle, ellipse, parabola, or hyperbola, and graph the equation
- **A-CED.1** Create equations and inequalities in one variable including ones with absolute value and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

- **CIRCLES**

- **G-GPE.3.1** Given a quadratic equation of the form  $ax^2 + by^2 + cx + dy + e = 0$ , use the method for completing the square to put the equation into standard form; identify whether the graph of the equation is a circle, ellipse, parabola, or hyperbola, and graph the equation

- **PARABOLAS**

- **G-GMD.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
- **A-CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

## 10. FUNCTIONS

- **DOMAIN AND RANGE**

- **F-IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

- **ARITHMETIC OPERATIONS ON FUNCTIONS**

- **F-BF.1.b** Combine standard function types using arithmetic operations.

- **MULTIPLE REPRESENTATIONS OF FUNCTIONS**

- **A-CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **F-IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

- **INVERSE FUNCTIONS**

- **F-BF.4.a** Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse.

## 11. PARENT FUNCTIONS AND TRANSFORMATIONS

- **PARENT FUNCTIONS**

- **F-IF.7.c** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- **F-BF.3** Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- **F-IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **F-IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- **F-IF.7.b** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- **F-IF.7.e** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

- **TRANSFORMATIONS OF PARENT FUNCTIONS**

- **F-IF.7.b** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- **F-IF.7.c** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- **F-IF.7.e** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- **F-BF.3** Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$

(both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

- **F-IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **F-IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

- **MULTIPLE TRANSFORMATIONS OF PARENT FUNCTIONS**

- **F-IF.7.c** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- **F-IF.7.e** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- **F-BF.3** Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- **F-IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **F-IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- **F-IF.7.b** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

- **ABSOLUTE VALUE FUNCTIONS**

- **F-IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- **F-IF.7.b** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

## 12. SYSTEMS OF EQUATIONS

- **SOLVING THREE-VARIABLE SYSTEMS OF LINEAR EQUATIONS**

- **A-CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

- **SYSTEMS OF NONLINEAR EQUATIONS**

- **A-REI.11** Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- **A-CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

## 13. SURFACE AREA

- **SURFACE AREA AND VOLUME OF SPHERES**

- **G-GMD.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
- **G-MG.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

- **SURFACE AREA OF COMPOSITE SOLIDS**

- **G-MG.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

- **SURFACE AREA OF SIMILAR SOLIDS**

- **G-MG.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

## 14. THREE-DIMENSIONAL GEOMETRY

### ● CONVERTING BETWEEN TWO-DIMENSIONAL FIGURES AND THREE-DIMENSIONAL SOLIDS

- **G-GMD.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

### ● MODELING SITUATIONS WITH GEOMETRY

- **G-MG.2** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
- **G-MG.3** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

## 15. STATISTICS AND PROBABILITY

### ● ANALYZING STATISTICAL SAMPLES

- **S-IC.1** Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
- **S-IC.4** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
- **S-IC.2** Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

### ● EXPERIMENTAL AND OBSERVATIONAL DESIGN

- **S-IC.3** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

### ● CONCLUSIONS IN DATA

- **S-IC.5** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

### ● NORMAL DISTRIBUTION

- **S-ID.4** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

### ● ANALYZING DECISIONS IN PROBABILITY

- **S-MD.6** Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
- **S-MD.7** Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).