

California Tutorials are designed specifically for the California Common Core State Standards and the California Next Generation Science Standards to prepare students for the Smarter Balanced Assessment Consortium (SBAC) exams and the California Science Tests.

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

1. RATIONAL AND IRRATIONAL NUMBERS

• OPERATIONS ON RATIONAL AND IRRATIONAL NUMBERS

- **N-RN.3** Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

• LAWS OF EXPONENTS

- **N-RN.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
- **N-RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.

2. EXPRESSIONS AND EQUATIONS

• FORMULATING AND SIMPLIFYING ALGEBRAIC EXPRESSIONS

- **F-BF.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **A-SSE.1.a** Interpret parts of an expression, such as terms, factors, and coefficients.
- **A-SSE.2** Use the structure of an expression to identify ways to rewrite it.

• ONE-STEP EQUATIONS AND INEQUALITIES

- **A-CED.1** Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.
- **F-BF.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.

• MULTI-STEP EQUATIONS AND INEQUALITIES

- **A-CED.1** Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.

- **LITERAL EQUATIONS**

- **A-CED.1** Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.
- **A-CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

3. FUNCTIONS

- **FUNCTIONS AND RELATIONS**

- **F-IF.7.b** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

- **DOMAIN AND RANGE**

- **F-IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

- **MULTIPLE REPRESENTATIONS OF FUNCTIONS**

- **F-IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- **A-CED.1** Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.
- **A-CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

4. POINTS, LINES, AND ANGLES

- **PARALLEL LINES AND ANGLE RELATIONSHIPS**

- **G-CO.9** Prove theorems about lines and angles.

- **PERPENDICULAR BISECTOR AND ANGLE BISECTOR THEOREMS**

- **G-CO.9** Prove theorems about lines and angles.
- **G-CO.10** Prove theorems about triangles.
- **G-SRT.4** Prove theorems about triangles.

5. THE COORDINATE PLANE

- **LENGTH AND THE DISTANCE FORMULA**

- **G-GPE.6** Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

- **MIDPOINT FORMULA ON THE COORDINATE PLANE**

- **G-GPE.6** Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

- **CONJECTURES IN COORDINATE GEOMETRY**

- **G-GPE.4** Use coordinates to prove simple geometric theorems algebraically.

6. CONIC SECTIONS

- **CIRCLES**

- **G-GPE.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

- **PARABOLAS**

- **A-CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **G-GPE.2** Derive the equation of a parabola given a focus and directrix.

7. GEOMETRIC TRANSFORMATIONS

- **TRANSFORMATIONS ON THE COORDINATE PLANE**

- **G-SRT.1.a** A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- **G-SRT.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
- **G-SRT.1.b** The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

- **DILATIONS, TRANSLATIONS, ROTATIONS, AND REFLECTIONS**

- **G-SRT.1.a** A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- **G-SRT.1.b** The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
- **G-SRT.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

8. CONGRUENCE AND SIMILARITY

- **TRIANGLES AND CONGRUENCE TRANSFORMATIONS**

- **G-CO.10** Prove theorems about triangles.
- **G-SRT.4** Prove theorems about triangles.
- **G-SRT.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

- **TRIANGLES AND SIMILARITY TRANSFORMATIONS**

- **G-SRT.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
- **G-SRT.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
- **G-CO.10** Prove theorems about triangles.
- **G-SRT.3** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
- **G-SRT.4** Prove theorems about triangles.

- **SIMILARITY OF OTHER POLYGONS**

- **G-SRT.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

9. TRIANGLES AND QUADRILATERALS

- **CLASSIFYING TRIANGLES**

- **G-CO.10** Prove theorems about triangles.

- **G-GMD.6** Verify experimentally that in a triangle, angles opposite longer sides are larger, sides opposite larger angles are longer, and the sum of any two side lengths is greater than the remaining side length; apply these relationships to solve real-world and mathematical problems.
- **G-SRT.4** Prove theorems about triangles.

- **TRIANGLE ANGLE THEOREMS**

- **G-CO.10** Prove theorems about triangles.
- **G-SRT.4** Prove theorems about triangles.

- **TRIANGLE BISECTORS**

- **G-CO.9** Prove theorems about lines and angles.
- **G-CO.10** Prove theorems about triangles.
- **G-SRT.4** Prove theorems about triangles.
- **G-SRT.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
- **G-C.3** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

- **MEDIANS AND ALTITUDES OF TRIANGLES**

- **G-CO.10** Prove theorems about triangles.
- **G-SRT.4** Prove theorems about triangles.

10. QUADRILATERALS AND CONSTRUCTIONS

- **PARALLELOGRAMS AND RECTANGLES**

- **G-CO.11** Prove theorems about parallelograms.

- **SQUARES AND RHOMBI**

- **G-CO.11** Prove theorems about parallelograms.

- **CONSTRUCTIONS**

- **G-C.4** Construct a tangent line from a point outside a given circle to the circle.

11. TRIANGLES AND TRIGONOMETRY

- **THE PYTHAGOREAN THEOREM**

- **G-CO.10** Prove theorems about triangles.
- **G-SRT.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
- **G-SRT.4** Prove theorems about triangles.
- **G-SRT.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

- **TRIGONOMETRIC RATIOS**

- **G-SRT.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
- **G-SRT.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
- **G-SRT.8.1** Derive and use the trigonometric ratios for special right triangles ($30^\circ, 60^\circ, 90^\circ$ and $45^\circ, 45^\circ, 90^\circ$).
- **G-SRT.7** Explain and use the relationship between the sine and cosine of complementary angles.

- **RADIANS AND THE UNIT CIRCLE**

- **G-C.5** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Convert between degrees and radians.
- **G-SRT.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
- **G-SRT.8.1** Derive and use the trigonometric ratios for special right triangles ($30^\circ, 60^\circ, 90^\circ$ and $45^\circ, 45^\circ, 90^\circ$).

- **TRIGONOMETRIC FUNCTIONS**

- **F-TF.8** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

12. CIRCLES

- **CIRCLE BASICS**

- **G-C.2** Identify and describe relationships among inscribed angles, radii, and chords.

- **CENTRAL ANGLES, INSCRIBED ANGLES, AND CHORDS**

- **G-C.2** Identify and describe relationships among inscribed angles, radii, and chords.
- **G-CO.9** Prove theorems about lines and angles.
- **G-C.3** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

- **SECANTS, ANGLES, AND INTERCEPTED ARCS**

- **G-CO.9** Prove theorems about lines and angles.
- **G-C.2** Identify and describe relationships among inscribed angles, radii, and chords.

- **TANGENTS, ANGLES, AND INTERCEPTED ARCS**

- **G-CO.9** Prove theorems about lines and angles.
- **G-C.2** Identify and describe relationships among inscribed angles, radii, and chords.

13. ADVANCED CIRCLE PROPERTIES

- **CONGRUENT AND SIMILAR CIRCLES**

- **G-C.1** Prove that all circles are similar.
- **G-SRT.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

- **CIRCUMFERENCE AND ARC LENGTH**

- **G-GMD.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

- **AREA OF CIRCLES AND SECTORS**

- **G-GMD.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
- **G-C.5** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Convert

between degrees and radians.

14. EXPONENTIAL FUNCTIONS, EQUATIONS, AND INEQUALITIES

• EXPONENTIAL FUNCTIONS

- **A-SSE.3.c** Use the properties of exponents to transform expressions for exponential functions.
- **F-IF.8.b** Use the properties of exponents to interpret expressions for exponential functions.
- **A-SSE.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **A-CED.1** Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.

• EXPONENTIAL GROWTH AND DECAY

- **F-IF.8.b** Use the properties of exponents to interpret expressions for exponential functions.
- **A-SSE.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **F-LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
- **A-CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

• SOLVING EXPONENTIAL INEQUALITIES

- **A-SSE.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **A-CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

15. POLYNOMIALS

• POLYNOMIAL BASICS

- **A-SSE.1.a** Interpret parts of an expression, such as terms, factors, and coefficients.

• ADDITION AND SUBTRACTION OF POLYNOMIALS

- **A-APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

• MULTIPLICATION OF POLYNOMIALS

- **A-APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

• ARITHMETIC OPERATIONS ON FUNCTIONS

- **F-BF.1.b** Combine standard function types using arithmetic operations.

16. FACTORING

• FACTORING QUADRATIC TRINOMIALS

- **A-SSE.3.a** Factor a quadratic expression to reveal the zeros of the function it defines.
- **A-REI.4.b** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b .

• FACTORING SPECIAL CASES

- **A-SSE.2** Use the structure of an expression to identify ways to rewrite it.
- **A-SSE.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity.

17. COMPLEX NUMBERS

• COMPLEX NUMBERS

- **N-CN.1** Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
- **N-CN.2** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

• POLYNOMIAL IDENTITIES AND COMPLEX NUMBERS

- **N-CN.8** Extend polynomial identities to the complex numbers.
- **A-SSE.2** Use the structure of an expression to identify ways to rewrite it.
- **N-CN.7** Solve quadratic equations with real coefficients that have complex solutions.
- **A-REI.4.b** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b .
- **N-CN.9** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

18. REPRESENTATIONS OF QUADRATIC FUNCTIONS

• QUADRATIC FUNCTIONS

- **F-IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **F-IF.7.a** Graph linear and quadratic functions and show intercepts, maxima, and minima.
- **F-LE.6** Apply quadratic functions to physical problems, such as the motion of an object under the force of gravity.

• ANALYZING GRAPHS OF QUADRATIC FUNCTIONS

- **F-IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- **F-IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **F-IF.7.a** Graph linear and quadratic functions and show intercepts, maxima, and minima.
- **F-IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

• REPRESENTATIONS OF QUADRATIC FUNCTIONS

- **A-SSE.2** Use the structure of an expression to identify ways to rewrite it.
- **A-REI.4.a** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
- **F-IF.8.a** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- **A-CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **A-SSE.3.a** Factor a quadratic expression to reveal the zeros of the function it defines.
- **F-IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **F-LE.6** Apply quadratic functions to physical problems, such as the motion of an object under the force of gravity.

- **QUADRATIC PARENT FUNCTION**

- **F-IF.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*

- **TRANSFORMATIONS OF THE QUADRATIC PARENT FUNCTION**

- **F-BF.3** *Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.*
- **F-IF.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*

19. SOLVING QUADRATIC EQUATIONS

- **SOLVING QUADRATIC FUNCTIONS WITH FACTORING**

- **A-SSE.3.a** *Factor a quadratic expression to reveal the zeros of the function it defines.*
- **A-REI.4.b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b .*
- **F-IF.8.a** *Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.*
- **F-LE.6** *Apply quadratic functions to physical problems, such as the motion of an object under the force of gravity.*
- **F-IF.7.a** *Graph linear and quadratic functions and show intercepts, maxima, and minima.*

- **COMPLETING THE SQUARE**

- **A-REI.4.a** *Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.*
- **A-REI.4.b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b .*
- **F-IF.8.a** *Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.*
- **A-SSE.3.b** *Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.*
- **F-IF.7.a** *Graph linear and quadratic functions and show intercepts, maxima, and minima.*

- **QUADRATIC FORMULA**

- **A-REI.4.b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b .*
- **F-IF.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
- **A-REI.4.a** *Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.*
- **F-BF.1.a** *Determine an explicit expression, a recursive process, or steps for calculation from a context.*

- **COMPLEX NUMBERS AND QUADRATIC FUNCTIONS**

- **A-REI.4.b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b .*
- **N-CN.7** *Solve quadratic equations with real coefficients that have complex solutions.*
- **N-CN.1** *Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.*

20. NONLINEAR FUNCTIONS

• INVERSE FUNCTIONS

- **F-BF.4.a** Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.

• ABSOLUTE VALUE FUNCTIONS

- **F-IF.7.b** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- **F-BF.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

• SYSTEMS OF NONLINEAR EQUATIONS

- **A-REI.7** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

21. VOLUME

• VOLUME OF PRISMS AND PYRAMIDS

- **G-GMD.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
- **G-GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

• VOLUME OF CYLINDERS AND CONES

- **G-GMD.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
- **G-GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

• VOLUME OF COMPOSITE SOLIDS

- **G-GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

• VOLUME OF SIMILAR SOLIDS

- **G-GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
- **G-GMD.5** Know that the effect of a scale factor k greater than zero on length, area, and volume is to multiply each by k , k^2 , and k^3 , respectively; determine length, area and volume measures using scale factors.

22. BASIC PROBABILITY CONCEPTS

• INTRODUCTION TO PROBABILITY

- **S-CP.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
- **S-CP.2** Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
- **S-CP.8** Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model.
- **S-CP.7** Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.

• COMBINATIONS AND PERMUTATIONS

- **S-CP.9** Use permutations and combinations to compute probabilities of compound events and solve problems.

23. ADVANCED PROBABILITY CONCEPTS

• **CONDITIONAL PROBABILITY**

- **S-CP.3** Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of given A is the same as the probability of B.
- **S-CP.5** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
- **S-CP.6** Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
- **S-CP.2** Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
- **S-CP.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

• **GEOMETRIC PROBABILITIES**

- **S-CP.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
- **S-MD.7** Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).
- **S-CP.7** Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.

• **ANALYZING DECISIONS IN PROBABILITY**

- **S-MD.6** Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
- **S-MD.7** Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).