Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

This Tutorial is built to state standards.

1. EXPRESSIONS, EQUATIONS, AND INEQUALITIES

- **FORMULATING AND SIMPLIFYING ALGEBRAIC EXPRESSIONS**
  - **CCSS.Math.Content.HSF-BF.A.1.a** *Determine an explicit expression, a recursive process, or steps for calculation from a context.*
  - **CCSS.Math.Content.HSA-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
  - **CCSS.Math.Content.HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*

- **LITERAL EQUATIONS**
  - **CCSS.Math.Content.HSA-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems.*
  - **CCSS.Math.Content.HSA-REI.B.3** *Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.*
  - **CCSS.Math.Content.HSA-CED.A.4** *Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.*

- **SUMS OF GEOMETRIC SEQUENCES**
  - **CCSS.Math.Content.HSA-SSE.B.4** *Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.*

2. LOGARITHMIC EXPRESSIONS AND FUNCTIONS

- **LOGARITHMIC FUNCTIONS**
  - **CCSS.Math.Content.HSF-BF.B.4.c** *Read values of an inverse function from a graph or a table, given that the function has an inverse.*
  - **CCSS.Math.Content.HSF-BF.B.5** *Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.*
  - **CCSS.Math.Content.HSF-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
  - **CCSS.Math.Content.HSF-LE.A.4** *For exponential models, express as a logarithm the solution to ab to the ct power = where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.*
  - **CCSS.Math.Content.HSF-IF.C.7.e** *Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.*

- **EVALUATING LOGARITHMIC EXPRESSIONS**
  - **CCSS.Math.Content.HSA-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
  - **CCSS.Math.Content.HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*

3. EXPONENTIAL AND LOGARITHMIC EQUATIONS

- **SOLVING EXPONENTIAL EQUATIONS**
  - **CCSS.Math.Content.HSA-SSE.B.3.c** *Use the properties of exponents to transform expressions for exponential functions.*
  - **CCSS.Math.Content.HSF-IF.C.8.b** *Use the properties of exponents to interpret expressions for exponential functions.*
  - **CCSS.Math.Content.HSF-IF.C.7.e** *Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.*

- **SOLVING LOGARITHMIC EQUATIONS**
  - **CCSS.Math.Content.HSF-IF.A.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = (x).*
  - **CCSS.Math.Content.HSF-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*

4. ARITHMETIC WITH POLYNOMIALS

- **POLYNOMIAL BASICS**
  - **CCSS.Math.Content.HSA-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
  - **CCSS.Math.Content.HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*

- **ADDITION AND SUBTRACTION OF POLYNOMIALS**
  - **CCSS.Math.Content.HSA-APR.A.1** *Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.*

- **MULTIPLICATION OF POLYNOMIALS**
  - **CCSS.Math.Content.HSA-APR.A.1** *Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.*

- **DIVISION OF POLYNOMIALS**
  - **CCSS.Math.Content.HSA-APR.B.2** *Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).*
  - **CCSS.Math.Content.HSA-APR.D.6** *Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form (x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.*
  - **CCSS.Math.Content.HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*

5. FACTORING AND GRAPHING WITH POLYNOMIALS

- **GRAPHS OF POLYNOMIAL FUNCTIONS**
  - **CCSS.Math.Content.HSA-APR.B.3** *Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.*
  - **CCSS.Math.Content.HSF-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*

- **CCSS.Math.Content.HSF-IF.C.7.c** *Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.*

- **FACTORING CUBIC POLYNOMIALS**
  - **CCSS.Math.Content.HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **CCSS.Math.Content.HSA-APR.C.4** *Prove polynomial identities and use them to describe numerical relationships.*

- **FACTORING HIGHER-ORDER POLYNOMIALS**
  - **CCSS.Math.Content.HSA-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
  - **CCSS.Math.Content.HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **CCSS.Math.Content.HSA-APR.C.4** *Prove polynomial identities and use them to describe numerical relationships.*

- **FACTOR THEOREM AND REMAINDER THEOREM**
  - **CCSS.Math.Content.HSF-IF.A.2** *Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.*
  - **CCSS.Math.Content.HSA-APR.B.2** *Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.*

6. POLYNOMIAL IDENTITIES

- **POLYNOMIAL IDENTITIES**
  - **CCSS.Math.Content.HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **CCSS.Math.Content.HSA-APR.C.4** *Prove polynomial identities and use them to describe numerical relationships.*
  - **CCSS.Math.Content.HSA-REI.B.4.a** *Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.*
  - **CCSS.Math.Content.HSA-REI.B.4.b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.*
  - **CCSS.Math.Content.HSA-APR.C.5** *Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.*

- **POLYNOMIAL IDENTITIES AND COMPLEX NUMBERS**
  - **CCSS.Math.Content.HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **CCSS.Math.Content.HSA-APR.C.4** *Prove polynomial identities and use them to describe numerical relationships.*
  - **CCSS.Math.Content.HSN-CN.C.8** *Extend polynomial identities to the complex numbers.*
  - **CCSS.Math.Content.HSA-REI.B.4.b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.*
  - **CCSS.Math.Content.HSN-CN.C.7** *Solve quadratic equations with real coefficients that have complex solutions.*
  - **CCSS.Math.Content.HSN-CN.C.9** *Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.*

7. RADICAL EXPRESSIONS, EQUATIONS, AND FUNCTIONS

- **ANALYZING GRAPHS OF SQUARE ROOT FUNCTIONS**
  - **CCSS.Math.Content.HSF-IF.C.7.b** *Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.*
  - **CCSS.Math.Content.HSF-BF.B.3** *Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.*
  - **CCSS.Math.Content.HSG-CO.B.6** *Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to*

decide if they are congruent.

- **CCSS.Math.Content.HSF-BF.B.4.c** *Read values of an inverse function from a graph or a table, given that the function has an inverse.*

- **SOLVING SQUARE ROOT EQUATIONS**
  - **CCSS.Math.Content.HSA-REI.A.2** *Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.*

8. RATIONAL EXPRESSIONS, EQUATIONS, AND FUNCTIONS

- **OPERATIONS WITH RATIONAL EXPRESSIONS**
  - **CCSS.Math.Content.HSA-APR.D.7** *Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.*
  - **CCSS.Math.Content.HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*

- **ANALYZING GRAPHS OF RATIONAL FUNCTIONS**
  - **CCSS.Math.Content.HSF-IF.C.7.d** *Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.*
  - **CCSS.Math.Content.HSF-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
  - **CCSS.Math.Content.HSF-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*

- **SOLVING RATIONAL EQUATIONS**
  - **CCSS.Math.Content.HSA-REI.A.2** *Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.*
  - **CCSS.Math.Content.HSF-IF.C.7.d** *Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.*

- **MODELING SITUATIONS WITH RATIONAL FUNCTIONS**

9. TRIGONOMETRY

- **RADIANS AND THE UNIT CIRCLE**
  - **CCSS.Math.Content.HSF-TF.A.1** *Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.*
  - **CCSS.Math.Content.HSF-TF.A.2** *Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.*
  - **CCSS.Math.Content.HSG-C.B.5** *Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.*
  - **CCSS.Math.Content.HSF-TF.A.3** *Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π–x, π+x, and 2π–x in terms of their values for x, where x is any real number.*
  - **CCSS.Math.Content.HSF-TF.A.4** *Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.*

- **TRIGONOMETRIC FUNCTIONS**
  - **CCSS.Math.Content.HSF-IF.C.7.e** *Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.*
  - **CCSS.Math.Content.HSF-TF.B.5** *Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*

- **CCSS.Math.Content.HSF-TF.C.8** *Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.*

- **LAWS OF SINE AND COSINE**
  - **CCSS.Math.Content.HSG-SRT.D.9** *Derive the formula $A = 1/2\ ab\ \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.*
  - **CCSS.Math.Content.HSG-SRT.D.10** *Prove the Laws of Sines and Cosines and use them to solve problems.*
  - **CCSS.Math.Content.HSG-SRT.D.11** *Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).*
  - **CCSS.Math.Content.HSG-SRT.C.8** *Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*

10. TRANSFORMING FUNCTIONS

- **DOMAIN AND RANGE**
  - **CCSS.Math.Content.HSF-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*

- **PARENT FUNCTIONS**
  - **CCSS.Math.Content.HSF-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
  - **CCSS.Math.Content.HSF-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
  - **CCSS.Math.Content.HSF-IF.C.7.b** *Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.*
  - **CCSS.Math.Content.HSF-IF.C.7.e** *Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.*

- **TRANSFORMATIONS OF PARENT FUNCTIONS**
  - **CCSS.Math.Content.HSF-BF.B.3** *Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and (x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.*
  - **CCSS.Math.Content.HSG-CO.B.6** *Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.*

- **MULTIPLE TRANSFORMATIONS OF PARENT FUNCTIONS**
  - **CCSS.Math.Content.HSF-BF.B.3** *Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and (x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.*
  - **CCSS.Math.Content.HSG-CO.B.6** *Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.*
  - **CCSS.Math.Content.HSG-CO.A.2** *Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).*

11. WORKING WITH FUNCTIONS

- **ARITHMETIC OPERATIONS ON FUNCTIONS**
  - **CCSS.Math.Content.HSF-BF.A.1.b** *Combine standard function types using arithmetic operations.*
  - **CCSS.Math.Content.HSA-APR.A.1** *Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.*

- **REPRESENTATIONS OF QUADRATIC FUNCTIONS**
  - **CCSS.Math.Content.HSA-REI.B.4.a** *Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.*
  - **CCSS.Math.Content.HSF-IF.C.8.a** *Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.*
  - **CCSS.Math.Content.HSF-IF.A.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).*
  - **CCSS.Math.Content.HSA-SSE.B.3.a** *Factor a quadratic expression to reveal the zeros of the function it defines.*
  - **CCSS.Math.Content.HSA-APR.B.3** *Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.*
  - **CCSS.Math.Content.HSF-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
  - **CCSS.Math.Content.HSF-IF.C.7.c** *Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.*

- **MULTIPLE REPRESENTATIONS OF FUNCTIONS**
  - **CCSS.Math.Content.HSF-IF.C.9** *Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).*
  - **CCSS.Math.Content.HSF-LE.A.1.a** *Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.*

- **INVERSE FUNCTIONS**
  - **CCSS.Math.Content.HSF-BF.B.4.a** *Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.*
  - **CCSS.Math.Content.HSF-BF.B.4.c** *Read values of an inverse function from a graph or a table, given that the function has an inverse.*
  - **CCSS.Math.Content.HSF-BF.B.4.b** *Verify by composition that one function is the inverse of another.*
  - **CCSS.Math.Content.HSF-BF.B.4.d** *Produce an invertible function from a non-invertible function by restricting the domain.*

12. SYSTEMS OF EQUATIONS

- **SOLVING THREE-VARIABLE SYSTEMS OF LINEAR EQUATIONS**
  - **CCSS.Math.Content.HSA-CED.A.3** *Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
  - **CCSS.Math.Content.HSA-REI.C.5** *Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.*

- **SYSTEMS OF NONLINEAR EQUATIONS**
  - **CCSS.Math.Content.HSA-REI.C.6** *Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.*
  - **CCSS.Math.Content.HSA-REI.C.7** *Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.*
  - **CCSS.Math.Content.HSA-REI.D.11** *Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*
  - **CCSS.Math.Content.HSA-REI.C.5** *Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.*

13. THREE-DIMENSIONAL GEOMETRY I

- **RELATING TWO-DIMENSIONAL FIGURES TO THREE-DIMENSIONAL SOLIDS**
  - **CCSS.Math.Content.HSG-GMD.B.4** *Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.*

- **SURFACE AREA AND VOLUME OF SPHERES**
  - **CCSS.Math.Content.HSG-GMD.A.2** *Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.*
  - **CCSS.Math.Content.HSG-GMD.B.4** *Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.*
  - **CCSS.Math.Content.HSG-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*
  - **CCSS.Math.Content.HSG-GMD.A.3** *Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*

- **SURFACE AREA OF COMPOSITE SOLIDS**
  - **CCSS.Math.Content.HSG-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

14. THREE-DIMENSIONAL GEOMETRY II

- **SURFACE AREA OF SIMILAR SOLIDS**
  - **CCSS.Math.Content.HSG-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

- **MODELING SITUATIONS WITH GEOMETRY**
  - **CCSS.Math.Content.HSG-MG.A.2** *Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*
  - **CCSS.Math.Content.HSG-MG.A.3** *Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*

15. STATISTICAL DESIGN AND ANALYSIS

- **ANALYZING STATISTICAL SAMPLES**
  - **CCSS.Math.Content.HSS-IC.A.1** *Understand statistics as a process for making inferences about population parameters based on a random sample from that population.*
  - **CCSS.Math.Content.HSS-IC.B.4** *Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.*
  - **CCSS.Math.Content.HSS-IC.A.2** *Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.*

- **EXPERIMENTAL AND OBSERVATIONAL DESIGN**
  - **CCSS.Math.Content.HSS-IC.B.3** *Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.*

- **CONCLUSIONS IN DATA**
  - **CCSS.Math.Content.HSS-IC.B.5** *Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.*
  - **CCSS.Math.Content.HSS-IC.B.6** *Evaluate reports based on data.*

16. STATISTICS AND PROBABILITY

- **NORMAL DISTRIBUTION**
  - **CCSS.Math.Content.HSS-ID.A.3** *Interpret differences in shape, center, and spread in the context of the data sets,*

accounting for possible effects of extreme data points (outliers).

- **CCSS.Math.Content.HSS-ID.A.4** *Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.*
- **CCSS.Math.Content.HSS-IC.B.4** *Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.*

- **ANALYZING DECISIONS IN PROBABILITY**
  - **CCSS.Math.Content.HSS-MD.B.6** *Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).*
  - **CCSS.Math.Content.HSS-MD.B.7** *Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).*