

NJSLA EOC Tutorials for New Jersey are designed specifically for the New Jersey Student Learning Standards to prepare students for the New Jersey Student Learning Assessment (NJSLA). EOC Categories are at the heart of NJSLA EOC Tutorial structure – bringing category-based learning to the student experience, and category-based performance and progress tracking to the teacher experience.

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

Test-Taking Strategies for EOC Tutorials allow students to practice and apply learning approaches that will hone their test-taking skills and focus them for success on the day of their EOC test.

## 1. AXIOMS OF EQUALITY

### • AXIOMS OF EQUALITY

- **HS-C.CCR** Solve multi-step mathematical problems requiring extended chains of reasoning and drawing on a synthesis of the knowledge and skills articulated across: 7-RP.A.3, 7-NS.A.3, 7-EE.B.3, 8-EE.C.7B, 8-EE.C.8c, N-RN.A.2, ASSE.A.1b, A-REI.A.1, A-REI.B.3, A-REI.B.4b, F-IF.A.2, F-IF.C.7a, F-IF.C.7e, G-SRT.B.5 and G-SRT.C.7.

## 2. LINEAR FUNCTIONS AND EQUATIONS

### • SLOPE

- **F-IF.4-2** For an exponential, polynomial, trigonometric, or logarithmic function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; end behavior; symmetries; and periodicity.
- **HS-D.2-10** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-BF.A, F-BF.3, F-IF.3, ACED.1, A-SSE.3, F-IF.B, F-IF.7.
- **F-IF.6-2** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval with functions limited to polynomial, exponential, logarithmic, and trigonometric functions.
- **F-IF.6-7** Estimate the rate of change from a graph.

### • GRAPHING AND MANIPULATING $Y = MX + B$

- **F-IF.4-2** For an exponential, polynomial, trigonometric, or logarithmic function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; end behavior; symmetries; and periodicity.
- **F-IF.6-2** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval with functions limited to polynomial, exponential, logarithmic, and trigonometric functions.
- **HS-D.2-7** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application

of course-level knowledge and skills articulated in A-CED, N-Q.2, A-SSE.3, AREI.6, A-REI.7, A-REI.12, A-REI.11-2.

- **HS-D.2-10** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-BF.A, F-BF.3, F-IF.3, ACED.1, A-SSE.3, F-IF.B, F-IF.7.

### 3. SYSTEMS OF LINEAR EQUATIONS

#### • SOLVING SYSTEMS OF LINEAR EQUATIONS: GRAPHING

- **HS-C.6.2** Base explanations/reasoning on the principle that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. Content Scope: A-REI.D

### 4. QUADRATIC EQUATIONS AND THE QUADRATIC FORMULA

#### • SOLVING QUADRATIC EQUATIONS BY FACTORING

- **A-Int.1** Solve equations that require seeing structure in expressions.
- **A-REI.4b-2** Solve quadratic equations in one variable. b) Recognize when the quadratic formula gives complex solutions.
- **HS-C.7.1** Base explanations/reasoning on the relationship between zeros and factors of polynomials. Content Scope: A-APR.B

#### • QUADRATIC FORMULA

- **A-Int.1** Solve equations that require seeing structure in expressions.
- **HS-C.3.1** Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about numbers or number systems. Content Scope: N-RN, N-CN
- **A-REI.4b-2** Solve quadratic equations in one variable. b) Recognize when the quadratic formula gives complex solutions.
- **N-CN.7** Solve quadratic equations with real coefficients that have complex solutions.

#### • REPRESENTATIONS OF QUADRATIC FUNCTIONS

- **HS-C.CCR** Solve multi-step mathematical problems requiring extended chains of reasoning and drawing on a synthesis of the knowledge and skills articulated across: 7-RP.A.3, 7-NS.A.3, 7-EE.B.3, 8-EE.C.7B, 8-EE.C.8c, N-RN.A.2, ASSE.A.1b, A-REI.A.1, A-REI.B.3, A-REI.B.4b, F-IF.A.2, F-IF.C.7a, F-IF.C.7e, G-SRT.B.5 and G-SRT.C.7.
- **F-IF.4-2** For an exponential, polynomial, trigonometric, or logarithmic function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; end behavior; symmetries; and periodicity.
- **HS-Int.3-3** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-LE, A-CED.1, A-SSE.3, F-IF.B, F-IF.7.
- **HS-C.6.4** Base explanations/reasoning on the principle that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. Content Scope: G-GPE.2

### 5. QUADRATIC FUNCTIONS

#### • COMPLETING THE SQUARE

- **HS-C.CCR** Solve multi-step mathematical problems requiring extended chains of reasoning and drawing on a synthesis of the knowledge and skills articulated across: 7-RP.A.3, 7-NS.A.3, 7-EE.B.3, 8-EE.C.7B, 8-EE.C.8c, N-RN.A.2, ASSE.A.1b, A-REI.A.1, A-REI.B.3, A-REI.B.4b, F-IF.A.2, F-IF.C.7a, F-IF.C.7e, G-SRT.B.5 and G-SRT.C.7.
- **HS-C.6.4** Base explanations/reasoning on the principle that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. Content Scope: G-GPE.2

#### • ANALYZING GRAPHS OF QUADRATIC FUNCTIONS

- **HS-C.CCR** Solve multi-step mathematical problems requiring extended chains of reasoning and drawing on a synthesis of the knowledge and skills articulated across: 7-RP.A.3, 7-NS.A.3, 7-EE.B.3, 8-EE.C.7B, 8-EE.C.8c, N-RN.A.2, ASSE.A.1b, A-REI.A.1, A-REI.B.3, A-REI.B.4b, F-IF.A.2, F-IF.C.7a, F-IF.C.7e, G-SRT.B.5 and G-SRT.C.7.
- **HS-C.6.4** Base explanations/reasoning on the principle that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. Content Scope: G-GPE.2

- **PARABOLAS**

- **HS-C.6.4** Base explanations/reasoning on the principle that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. Content Scope: G-GPE.2

## 6. LAWS OF EXPONENTS

- **LAWS OF EXPONENTS**

- **HS-C.18.4** Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about polynomials, rational expressions, or rational exponents. Content scope: N-RN, A-APR.(2, 3, 4, 6)
- **HS-C.3.2** Base explanations/reasoning on the properties of exponents. Content Scope: N-RN.A
- **N-RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.
- **HS-C.12.2** Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about functions. Content scope: F-IF.8b
- **HS-C.3.1** Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about numbers or number systems. Content Scope: N-RN, N-CN

## 7. EXPONENTIAL AND LOGARITHMIC FUNCTIONS

- **EXPONENTIAL FUNCTIONS**

- **A-SSE.3c-2** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression, where exponentials are limited to rational or real exponents. c) Use the properties of exponents to transform expressions for exponential functions. For example, the expression can be rewritten as  $(1.15^{1/12})^{12} \approx 1.012^{12}$  to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.
- **F-Int.1-2** Given a verbal description of a polynomial, exponential, trigonometric, or logarithmic functional dependence, write an expression for the function and demonstrate various knowledge and skills articulated in the Functions category in relation to this function.
- **F-IF.8b** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b) Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as  $y = (1.02)^t$ ,  $y = (0.97)^t$ ,  $y = (1.01)^{12t}$ ,  $y = (1.2)^t/10$ , and classify them as representing exponential growth or decay.
- **F-IF.6-2** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval with functions limited to polynomial, exponential, logarithmic, and trigonometric functions.
- **HS-C.CCR** Solve multi-step mathematical problems requiring extended chains of reasoning and drawing on a synthesis of the knowledge and skills articulated across: 7-RP.A.3, 7-NS.A.3, 7-EE.B.3, 8-EE.C.7B, 8-EE.C.8c, N-RN.A.2, ASSE.A.1b, A-REI.A.1, A-REI.B.3, A-REI.B.4b, F-IF.A.2, F-IF.C.7a, F-IF.C.7e, G-SRT.B.5 and G-SRT.C.7.
- **HS-D.2-10** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-BF.A, F-BF.3, F-IF.3, ACED.1, A-SSE.3, F-IF.B, F-IF.7.
- **HS-Int.3-3** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-LE, A-CED.1, A-SSE.3, F-IF.B, F-IF.7.
- **F-LE.2-3** Solve multi-step contextual problems with degree of difficulty appropriate to the course by constructing linear and/or exponential function models.
- **F-IF.4-2** For an exponential, polynomial, trigonometric, or logarithmic function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; end behavior; symmetries; and periodicity.
- **F-IF.7e-1** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. e) Graph exponential functions, showing intercepts and end behavior.
- **HS-C.12.2** Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about functions. Content scope: F-IF.8b

- **LOGARITHMIC FUNCTIONS**

- **HS-D.2-7** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in A-CED, N-Q.2, A-SSE.3, A-REI.6, A-REI.7, A-REI.12, A-REI.11-2.
- **HS-C.CCR** Solve multi-step mathematical problems requiring extended chains of reasoning and drawing on a synthesis of the knowledge and skills articulated across: 7-RP.A.3, 7-NS.A.3, 7-EE.B.3, 8-EE.C.7B, 8-EE.C.8c, N-RN.A.2, ASSE.A.1b, A-REI.A.1, A-REI.B.3, A-REI.B.4b, F-IF.A.2, F-IF.C.7a, F-IF.C.7e, G-SRT.B.5 and G-SRT.C.7.

- **A-REI.11-2** Find the solutions of where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect, e.g., using technology to graph the functions, make tables of values or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, quadratic, polynomial, rational, absolute value, exponential, and/or logarithmic functions.
- **HS-C.5.11** Given an equation or system of equations, reason about the number or nature of the solutions. Content Scope: A-REI.11, involving any of the function types measured in the standards.
- **F-Int.1-2** Given a verbal description of a polynomial, exponential, trigonometric, or logarithmic functional dependence, write an expression for the function and demonstrate various knowledge and skills articulated in the Functions category in relation to this function.
- **HS-D.2-10** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-BF.A, F-BF.3, F-IF.3, ACED.1, A-SSE.3, F-IF.B, F-IF.7.
- **HS-Int.3-3** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-LE, A-CED.1, A-SSE.3, F-IF.B, F-IF.7.
- **HS-C.6.2** Base explanations/reasoning on the principle that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. Content Scope: A-REI.D
- **F-IF.4-2** For an exponential, polynomial, trigonometric, or logarithmic function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; end behavior; symmetries; and periodicity.
- **F-IF.7e-2** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. e) Graph logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

## 8. ARITHMETIC OPERATIONS ON FUNCTIONS

### ● ARITHMETIC OPERATIONS ON FUNCTIONS

- **F-BF.1b-1** Represent arithmetic combinations of standard function types algebraically.
- **HS-D.2-10** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-BF.A, F-BF.3, F-IF.3, ACED.1, A-SSE.3, F-IF.B, F-IF.7.

## 9. INVERSE FUNCTIONS

### ● INVERSE FUNCTIONS

- **F-BF.Int.2** Find inverse functions to solve contextual problems. a) Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse. For example,  $f(x) = 2x^3$  or  $f(x) = x + 1/x - 1$  for  $x \neq 1$ .
- **HS-D.2-4** Solve multi-step contextual problems with degree of difficulty appropriate to the course that require writing an expression for an inverse function, as articulated in F.BF.4a.

## 10. EXPONENTIAL EQUATIONS, GROWTH, AND DECAY

### ● SOLVING EXPONENTIAL EQUATIONS

- **HS-D.2-7** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in A-CED, N-Q.2, A-SSE.3, AREI.6, A-REI.7, A-REI.12, A-REI.11-2.
- **HS-D.2-10** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-BF.A, F-BF.3, F-IF.3, ACED.1, A-SSE.3, F-IF.B, F-IF.7.
- **HS-Int.3-3** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-LE, A-CED.1, A-SSE.3, F-IF.B, F-IF.7.
- **A-Int.1** Solve equations that require seeing structure in expressions.
- **HS-C.3.2** Base explanations/reasoning on the properties of exponents. Content Scope: N-RN.A
- **F-IF.8b** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b) Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as  $y = (1.02)^t$ ,  $y = (0.97)^t$ ,  $y = (1.01)^{12t}$ ,  $y = (1.2)^t/10$ , and classify them as representing exponential growth or decay.
- **F-IF.4-2** For an exponential, polynomial, trigonometric, or logarithmic function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; end behavior; symmetries; and periodicity.

- **F-IF.7e-1** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. e) Graph exponential functions, showing intercepts and end behavior.

## ● EXPONENTIAL GROWTH AND DECAY

- **HS-D.2-10** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-BF.A, F-BF.3, F-IF.3, ACED.1, A-SSE.3, F-IF.B, F-IF.7.
- **HS-C.12.2** Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about functions. Content scope: F-IF.8b
- **F-IF.8b** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b) Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as  $y = (1.02)^t$ ,  $y = (0.97)^t$ ,  $y = (1.01)^{12t}$ ,  $y = (1.2)^t/10$ , and classify them as representing exponential growth or decay.
- **F-IF.6-2** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval with functions limited to polynomial, exponential, logarithmic, and trigonometric functions.
- **A-SSE.3c-2** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression, where exponentials are limited to rational or real exponents. c) Use the properties of exponents to transform expressions for exponential functions. For example, the expression can be rewritten as  $(1.15^{1/12})^{12} \approx 1.012^{12}$  to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

## 11. EVALUATING LOGARITHMIC EXPRESSIONS

### ● EVALUATING LOGARITHMIC EXPRESSIONS

- **F-Int.1-2** Given a verbal description of a polynomial, exponential, trigonometric, or logarithmic functional dependence, write an expression for the function and demonstrate various knowledge and skills articulated in the Functions category in relation to this function.
- **F-BF.Int.2** Find inverse functions to solve contextual problems. a) Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse. For example,  $f(x) = 2x^3$  or  $f(x) = x + 1/x - 1$  for  $x \neq 1$ .

## 12. SOLVING LOGARITHMIC EQUATIONS

### ● SOLVING LOGARITHMIC EQUATIONS

- **HS-D.2-7** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in A-CED, N-Q.2, A-SSE.3, AREI.6, A-REI.7, A-REI.12, A-REI.11-2.
- **HS-D.2-10** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-BF.A, F-BF.3, F-IF.3, ACED.1, A-SSE.3, F-IF.B, F-IF.7.
- **A-Int.1** Solve equations that require seeing structure in expressions.

## 13. COMPARING FUNCTIONS

### ● MULTIPLE REPRESENTATIONS OF FUNCTIONS

- **F-IF.9-2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Function types are limited to polynomial, exponential, logarithmic, and trigonometric functions.

## 14. SEQUENCES 1

### ● SEQUENCES

- **HS-D.2-10** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-BF.A, F-BF.3, F-IF.3, ACED.1, A-SSE.3, F-IF.B, F-IF.7.
- **F-BF.2** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

### ● ARITHMETIC AND GEOMETRIC SEQUENCES

- **F-BF.2** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations,

and translate between the two forms.

- **HS-D.2-10** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-BF.A, F-BF.3, F-IF.3, ACED.1, A-SSE.3, F-IF.B, F-IF.7.

## 15. SEQUENCES 2

### • SUMS OF GEOMETRIC SEQUENCES

- **HS-C.4.1** Derive and use a formula. Content Scope: A-SSE.4
- **HS-C.CCR** Solve multi-step mathematical problems requiring extended chains of reasoning and drawing on a synthesis of the knowledge and skills articulated across: 7-RP.A.3, 7-NS.A.3, 7-EE.B.3, 8-EE.C.7B, 8-EE.C.8c, N-RN.A.2, ASSE.A.1b, A-REI.A.1, A-REI.B.3, A-REI.B.4b, F-IF.A.2, F-IF.C.7a, F-IF.C.7e, G-SRT.B.5 and G-SRT.C.7.
- **A-SSE.4-2** Use the formula for the sum of a finite geometric series to solve multistep contextual problems.

## 16. OPERATIONS WITH POLYNOMIALS 1

### • ADDITION AND SUBTRACTION OF POLYNOMIALS

- **A-SSE.2-3** Use the structure of polynomial, rational or exponential expressions to identify ways to rewrite it. For example, see  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ .
- **HS-D.2-7** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in A-CED, N-Q.2, A-SSE.3, AREI.6, A-REI.7, A-REI.12, A-REI.11-2.
- **HS-C.CCR** Solve multi-step mathematical problems requiring extended chains of reasoning and drawing on a synthesis of the knowledge and skills articulated across: 7-RP.A.3, 7-NS.A.3, 7-EE.B.3, 8-EE.C.7B, 8-EE.C.8c, N-RN.A.2, ASSE.A.1b, A-REI.A.1, A-REI.B.3, A-REI.B.4b, F-IF.A.2, F-IF.C.7a, F-IF.C.7e, G-SRT.B.5 and G-SRT.C.7.

### • MULTIPLICATION OF POLYNOMIALS

- **A-SSE.2-3** Use the structure of polynomial, rational or exponential expressions to identify ways to rewrite it. For example, see  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ .
- **HS-D.2-7** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in A-CED, N-Q.2, A-SSE.3, AREI.6, A-REI.7, A-REI.12, A-REI.11-2.
- **HS-C.CCR** Solve multi-step mathematical problems requiring extended chains of reasoning and drawing on a synthesis of the knowledge and skills articulated across: 7-RP.A.3, 7-NS.A.3, 7-EE.B.3, 8-EE.C.7B, 8-EE.C.8c, N-RN.A.2, ASSE.A.1b, A-REI.A.1, A-REI.B.3, A-REI.B.4b, F-IF.A.2, F-IF.C.7a, F-IF.C.7e, G-SRT.B.5 and G-SRT.C.7.

## 17. OPERATIONS WITH POLYNOMIALS 2

### • DIVISION OF POLYNOMIALS

- **A-SSE.2-3** Use the structure of polynomial, rational or exponential expressions to identify ways to rewrite it. For example, see  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ .
- **A-APR.6** Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$  where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.
- **HS-D.2-7** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in A-CED, N-Q.2, A-SSE.3, AREI.6, A-REI.7, A-REI.12, A-REI.11-2.
- **HS-C.CCR** Solve multi-step mathematical problems requiring extended chains of reasoning and drawing on a synthesis of the knowledge and skills articulated across: 7-RP.A.3, 7-NS.A.3, 7-EE.B.3, 8-EE.C.7B, 8-EE.C.8c, N-RN.A.2, ASSE.A.1b, A-REI.A.1, A-REI.B.3, A-REI.B.4b, F-IF.A.2, F-IF.C.7a, F-IF.C.7e, G-SRT.B.5 and G-SRT.C.7.

## 18. FACTORING POLYNOMIALS

### • FACTORING CUBIC POLYNOMIALS

- **A-SSE.2-3** Use the structure of polynomial, rational or exponential expressions to identify ways to rewrite it. For example, see  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ .
- **A-SSE.2-6** Use the structure of a polynomial, rational, or exponential expression to rewrite it, in a case where two or more rewriting steps are required.
- **HS-C.7.1** Base explanations/reasoning on the relationship between zeros and factors of polynomials. Content Scope: A-

#### APR.B

- **HS-D.2-7** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in A-CED, N-Q.2, A-SSE.3, AREI.6, A-REI.7, A-REI.12, A-REI.11-2.
- **HS-C.CCR** Solve multi-step mathematical problems requiring extended chains of reasoning and drawing on a synthesis of the knowledge and skills articulated across: 7-RP.A.3, 7-NS.A.3, 7-EE.B.3, 8-EE.C.7B, 8-EE.C.8c, N-RN.A.2, ASSE.A.1b, A-REI.A.1, A-REI.B.3, A-REI.B.4b, F-IF.A.2, F-IF.C.7a, F-IF.C.7e, G-SRT.B.5 and G-SRT.C.7.

### ● FACTORING SPECIAL CASES

- **A-SSE.2-3** Use the structure of polynomial, rational or exponential expressions to identify ways to rewrite it. For example, see  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ .
- **A-SSE.2-6** Use the structure of a polynomial, rational, or exponential expression to rewrite it, in a case where two or more rewriting steps are required.
- **HS-D.2-7** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in A-CED, N-Q.2, A-SSE.3, AREI.6, A-REI.7, A-REI.12, A-REI.11-2.
- **HS-C.CCR** Solve multi-step mathematical problems requiring extended chains of reasoning and drawing on a synthesis of the knowledge and skills articulated across: 7-RP.A.3, 7-NS.A.3, 7-EE.B.3, 8-EE.C.7B, 8-EE.C.8c, N-RN.A.2, ASSE.A.1b, A-REI.A.1, A-REI.B.3, A-REI.B.4b, F-IF.A.2, F-IF.C.7a, F-IF.C.7e, G-SRT.B.5 and G-SRT.C.7.

### ● FACTORING HIGHER-ORDER POLYNOMIALS

- **A-SSE.2-3** Use the structure of polynomial, rational or exponential expressions to identify ways to rewrite it. For example, see  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ .
- **A-SSE.2-6** Use the structure of a polynomial, rational, or exponential expression to rewrite it, in a case where two or more rewriting steps are required.
- **HS-C.7.1** Base explanations/reasoning on the relationship between zeros and factors of polynomials. Content Scope: A-APR.B
- **HS-D.2-7** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in A-CED, N-Q.2, A-SSE.3, AREI.6, A-REI.7, A-REI.12, A-REI.11-2.
- **HS-C.CCR** Solve multi-step mathematical problems requiring extended chains of reasoning and drawing on a synthesis of the knowledge and skills articulated across: 7-RP.A.3, 7-NS.A.3, 7-EE.B.3, 8-EE.C.7B, 8-EE.C.8c, N-RN.A.2, ASSE.A.1b, A-REI.A.1, A-REI.B.3, A-REI.B.4b, F-IF.A.2, F-IF.C.7a, F-IF.C.7e, G-SRT.B.5 and G-SRT.C.7.
- **HS-C.18.4** Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about polynomials, rational expressions, or rational exponents. Content scope: N-RN, A-APR.(2, 3, 4, 6)

### ● POLYNOMIAL IDENTITIES

- **HS-C.18.4** Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about polynomials, rational expressions, or rational exponents. Content scope: N-RN, A-APR.(2, 3, 4, 6)
- **HS-C.8.2** Construct, autonomously, chains of reasoning that will justify or refute algebraic propositions or conjectures. Content Scope: A-APR.4
- **HS-C.8.3** Construct, autonomously, chains of reasoning that will justify or refute algebraic propositions or conjectures. Content Scope: A-APR
- **A-SSE.2-3** Use the structure of polynomial, rational or exponential expressions to identify ways to rewrite it. For example, see  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ .
- **A-SSE.2-6** Use the structure of a polynomial, rational, or exponential expression to rewrite it, in a case where two or more rewriting steps are required.
- **HS-D.2-7** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in A-CED, N-Q.2, A-SSE.3, AREI.6, A-REI.7, A-REI.12, A-REI.11-2.
- **HS-C.CCR** Solve multi-step mathematical problems requiring extended chains of reasoning and drawing on a synthesis of the knowledge and skills articulated across: 7-RP.A.3, 7-NS.A.3, 7-EE.B.3, 8-EE.C.7B, 8-EE.C.8c, N-RN.A.2, ASSE.A.1b, A-REI.A.1, A-REI.B.3, A-REI.B.4b, F-IF.A.2, F-IF.C.7a, F-IF.C.7e, G-SRT.B.5 and G-SRT.C.7.

## 19. FACTOR THEOREM AND REMAINDER THEOREM

### ● FACTOR THEOREM AND REMAINDER THEOREM

- **HS-C.7.1** Base explanations/reasoning on the relationship between zeros and factors of polynomials. Content Scope: A-APR.B

- **HS-C.18.4** Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about polynomials, rational expressions, or rational exponents. Content scope: N-RN, A-APR.(2, 3, 4, 6)
- **A-APR.6** Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$  where  $a(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.
- **HS-C.8.3** Construct, autonomously, chains of reasoning that will justify or refute algebraic propositions or conjectures. Content Scope: A-APR
- **A-APR.2** Know and apply the Remainder Theorem: For a polynomial  $p(x)$  and a number  $a$ , the remainder on division by  $x - a$  is  $p(a)$ , so  $p(a) = 0$  if and only if  $(x - a)$  is a factor of  $p(x)$ .

## 20. COMPLEX NUMBERS

### ● COMPLEX NUMBERS

- **HS-C.3.1** Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about numbers or number systems. Content Scope: N-RN, N-CN
- **N-CN.1** Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.
- **N-CN.2** Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

### ● COMPLEX NUMBERS AND QUADRATIC FUNCTIONS

- **HS-C.3.1** Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about numbers or number systems. Content Scope: N-RN, N-CN
- **A-REI.4b-2** Solve quadratic equations in one variable. b) Recognize when the quadratic formula gives complex solutions.
- **N-CN.1** Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.
- **N-CN.7** Solve quadratic equations with real coefficients that have complex solutions.
- **N-CN.2** Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

## 21. POLYNOMIAL FUNCTIONS AND COMPLEX NUMBERS

### ● GRAPHS OF POLYNOMIAL FUNCTIONS

- **F-IF.4-2** For an exponential, polynomial, trigonometric, or logarithmic function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; end behavior; symmetries; and periodicity.
- **HS-C.18.4** Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about polynomials, rational expressions, or rational exponents. Content scope: N-RN, A-APR.(2, 3, 4, 6)
- **HS-D.2-10** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-BF.A, F-BF.3, F-IF.3, ACED.1, A-SSE.3, F-IF.B, F-IF.7.
- **HS-Int.3-3** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-LE, A-CED.1, A-SSE.3, F-IF.B, F-IF.7.
- **F-IF.7c** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. c) Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

### ● POLYNOMIAL IDENTITIES AND COMPLEX NUMBERS

- **A-SSE.2-3** Use the structure of polynomial, rational or exponential expressions to identify ways to rewrite it. For example, see  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ .
- **A-SSE.2-6** Use the structure of a polynomial, rational, or exponential expression to rewrite it, in a case where two or more rewriting steps are required.
- **HS-D.2-7** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in A-CED, N-Q.2, A-SSE.3, AREI.6, A-REI.7, A-REI.12, A-REI.11-2.
- **HS-C.CCR** Solve multi-step mathematical problems requiring extended chains of reasoning and drawing on a synthesis of the knowledge and skills articulated across: 7-RP.A.3, 7-NS.A.3, 7-EE.B.3, 8-EE.C.7B, 8-EE.C.8c, N-RN.A.2, ASSE.A.1b, A-



REI.A.1, A-REI.B.3, A-REI.B.4b, F-IF.A.2, F-IF.C.7a, F-IF.C.7e, G-SRT.B.5 and G-SRT.C.7.

- **HS-C.18.4** Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about polynomials, rational expressions, or rational exponents. Content scope: N-RN, A-APR.(2, 3, 4, 6)
- **HS-C.3.1** Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about numbers or number systems. Content Scope: N-RN, N-CN
- **A-REI.4b-2** Solve quadratic equations in one variable. b) Recognize when the quadratic formula gives complex solutions.
- **N-CN.7** Solve quadratic equations with real coefficients that have complex solutions.

## 22. RADICAL AND RATIONAL EQUATIONS

### ● OPERATIONS WITH RATIONAL EXPRESSIONS

- **A-SSE.2-3** Use the structure of polynomial, rational or exponential expressions to identify ways to rewrite it. For example, see  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ .
- **A-SSE.2-6** Use the structure of a polynomial, rational, or exponential expression to rewrite it, in a case where two or more rewriting steps are required.
- **HS-C.3.1** Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about numbers or number systems. Content Scope: N-RN, N-CN
- **HS-C.3.2** Base explanations/reasoning on the properties of exponents. Content Scope: N-RN.A
- **N-RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.

### ● SOLVING RATIONAL EQUATIONS

- **HS-D.2-7** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in A-CED, N-Q.2, A-SSE.3, AREI.6, A-REI.7, A-REI.12, A-REI.11-2.
- **HS-C.CCR** Solve multi-step mathematical problems requiring extended chains of reasoning and drawing on a synthesis of the knowledge and skills articulated across: 7-RP.A.3, 7-NS.A.3, 7-EE.B.3, 8-EE.C.7B, 8-EE.C.8c, N-RN.A.2, ASSE.A.1b, A-REI.A.1, A-REI.B.3, A-REI.B.4b, F-IF.A.2, F-IF.C.7a, F-IF.C.7e, G-SRT.B.5 and G-SRT.C.7.
- **HS-C.18.4** Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about polynomials, rational expressions, or rational exponents. Content scope: N-RN, A-APR.(2, 3, 4, 6)
- **HS-C.16.3** Given an equation or system of equations, present the solution steps as a logical argument that concludes with the set of solutions (if any). Tasks are limited to simple rational or radical equations. Content scope: A-REI.1
- **A-REI.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- **HS-C.5.4** Given an equation or system of equations, reason about the number or nature of the solutions. Content Scope: A-REI.2

### ● MODELING SITUATIONS WITH RATIONAL FUNCTIONS

- **HS-D.2-7** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in A-CED, N-Q.2, A-SSE.3, AREI.6, A-REI.7, A-REI.12, A-REI.11-2.
- **HS-C.CCR** Solve multi-step mathematical problems requiring extended chains of reasoning and drawing on a synthesis of the knowledge and skills articulated across: 7-RP.A.3, 7-NS.A.3, 7-EE.B.3, 8-EE.C.7B, 8-EE.C.8c, N-RN.A.2, ASSE.A.1b, A-REI.A.1, A-REI.B.3, A-REI.B.4b, F-IF.A.2, F-IF.C.7a, F-IF.C.7e, G-SRT.B.5 and G-SRT.C.7.
- **HS-C.5.4** Given an equation or system of equations, reason about the number or nature of the solutions. Content Scope: A-REI.2
- **HS-C.18.4** Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about polynomials, rational expressions, or rational exponents. Content scope: N-RN, A-APR.(2, 3, 4, 6)
- **HS-C.16.3** Given an equation or system of equations, present the solution steps as a logical argument that concludes with the set of solutions (if any). Tasks are limited to simple rational or radical equations. Content scope: A-REI.1
- **A-REI.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

### ● SOLVING SQUARE ROOT EQUATIONS

- **HS-D.2-7** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in A-CED, N-Q.2, A-SSE.3, AREI.6, A-REI.7, A-REI.12, A-REI.11-2.
- **HS-C.16.3** Given an equation or system of equations, present the solution steps as a logical argument that concludes with the set of solutions (if any). Tasks are limited to simple rational or radical equations. Content scope: A-REI.1

- **A-REI.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- **HS-C.5.4** Given an equation or system of equations, reason about the number or nature of the solutions. Content Scope: A-REI.2

## 23. GRAPHS OF RADICAL AND RATIONAL FUNCTIONS

### ● ANALYZING GRAPHS OF SQUARE ROOT FUNCTIONS

- **F-IF.4-2** For an exponential, polynomial, trigonometric, or logarithmic function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; end behavior; symmetries; and periodicity.
- **HS-D.2-10** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-BF.A, F-BF.3, F-IF.3, ACED.1, A-SSE.3, F-IF.B, F-IF.7.
- **HS-Int.3-3** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-LE, A-CED.1, A-SSE.3, F-IF.B, F-IF.7.
- **HS-D.2-4** Solve multi-step contextual problems with degree of difficulty appropriate to the course that require writing an expression for an inverse function, as articulated in F.BF.4a.
- **F-BF.3-2** Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $kf(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs, limiting the function types to polynomial, exponential, logarithmic, and trigonometric functions.

### ● ANALYZING GRAPHS OF RATIONAL FUNCTIONS

- **F-IF.4-2** For an exponential, polynomial, trigonometric, or logarithmic function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; end behavior; symmetries; and periodicity.
- **F-BF.3-2** Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $kf(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs, limiting the function types to polynomial, exponential, logarithmic, and trigonometric functions.
- **HS-D.2-10** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-BF.A, F-BF.3, F-IF.3, ACED.1, A-SSE.3, F-IF.B, F-IF.7.
- **HS-Int.3-3** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-LE, A-CED.1, A-SSE.3, F-IF.B, F-IF.7.

## 24. NONLINEAR EQUATIONS AND FUNCTIONS

### ● SYSTEMS OF NONLINEAR EQUATIONS

- **HS-D.2-7** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in A-CED, N-Q.2, A-SSE.3, A-REI.6, A-REI.7, A-REI.12, A-REI.11-2.
- **A-REI.7** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line  $y = -3x$  and the circle  $x^2 + y^2 = 3$ .
- **A-REI.11-2** Find the solutions of where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect, e.g., using technology to graph the functions, make tables of values or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, quadratic, polynomial, rational, absolute value, exponential, and/or logarithmic functions.
- **HS-C.5.11** Given an equation or system of equations, reason about the number or nature of the solutions. Content Scope: A-REI.11, involving any of the function types measured in the standards.

### ● LINEAR VERSUS NONLINEAR FUNCTIONS

- **F-IF.6-2** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval with functions limited to polynomial, exponential, logarithmic, and trigonometric functions.
- **HS-C.CCR** Solve multi-step mathematical problems requiring extended chains of reasoning and drawing on a synthesis of the knowledge and skills articulated across: 7-RP.A.3, 7-NS.A.3, 7-EE.B.3, 8-EE.C.7B, 8-EE.C.8c, N-RN.A.2, ASSE.A.1b, A-REI.A.1, A-REI.B.3, A-REI.B.4b, F-IF.A.2, F-IF.C.7a, F-IF.C.7e, G-SRT.B.5 and G-SRT.C.7.
- **HS-D.2-10** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-BF.A, F-BF.3, F-IF.3, ACED.1, A-SSE.3, F-IF.B, F-IF.7.

- **HS-Int.3-3** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-LE, A-CED.1, A-SSE.3, F-IF.B, F-IF.7.
- **F-LE.2-3** Solve multi-step contextual problems with degree of difficulty appropriate to the course by constructing linear and/or exponential function models.
- **F-IF.4-2** For an exponential, polynomial, trigonometric, or logarithmic function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; end behavior; symmetries; and periodicity.

## 25. TRIGONOMETRIC FUNCTIONS

### ● TRIGONOMETRIC FUNCTIONS

- **F-IF.4-2** For an exponential, polynomial, trigonometric, or logarithmic function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; end behavior; symmetries; and periodicity.
- **F-BF.3-2** Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $kf(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs, limiting the function types to polynomial, exponential, logarithmic, and trigonometric functions.
- **F-Int.1-2** Given a verbal description of a polynomial, exponential, trigonometric, or logarithmic functional dependence, write an expression for the function and demonstrate various knowledge and skills articulated in the Functions category in relation to this function.
- **HS-D.2-10** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-BF.A, F-BF.3, F-IF.3, ACED.1, A-SSE.3, F-IF.B, F-IF.7.
- **HS-Int.3-3** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-LE, A-CED.1, A-SSE.3, F-IF.B, F-IF.7.
- **F-IF.7e-2** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. e) Graph logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- **F-Int.3** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-TF.5, F-IF.B, F-IF.7, limited to trigonometric functions.
- **HS-C.11.1** Express reasoning about trigonometric functions and the unit circle. Content scope: F-TF.2, F-TF.8
- **F-TF.8-2** Use the Pythagorean identity  $\sin^2 \theta + \cos^2 \theta = 1$  to find  $\sin \theta$ ,  $\cos \theta$ , or  $\tan \theta$ , given  $\sin \theta$ ,  $\cos \theta$ , or  $\tan \theta$ , and the quadrant of the angle.

## 26. RADIANs AND THE UNIT CIRCLE

### ● RADIANs AND THE UNIT CIRCLE

- **F-TF.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- **HS-C.11.1** Express reasoning about trigonometric functions and the unit circle. Content scope: F-TF.2, F-TF.8

## 27. PARENT FUNCTIONS AND TRANSFORMATIONS

### ● PARENT FUNCTIONS

- **F-IF.4-2** For an exponential, polynomial, trigonometric, or logarithmic function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; end behavior; symmetries; and periodicity.
- **HS-D.2-10** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-BF.A, F-BF.3, F-IF.3, ACED.1, A-SSE.3, F-IF.B, F-IF.7.
- **HS-Int.3-3** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-LE, A-CED.1, A-SSE.3, F-IF.B, F-IF.7.
- **HS-C.9.2** Express reasoning about transformations of functions. Content scope: F-BF.3, which may involve polynomial, exponential, logarithmic, or trigonometric functions. Tasks also may involve even and odd functions.
- **F-IF.7e-1** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. e) Graph exponential functions, showing intercepts and end behavior.
- **F-IF.7e-2** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using

technology for more complicated cases. e) Graph logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

- **F-BF.3-2** Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $kf(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs, limiting the function types to polynomial, exponential, logarithmic, and trigonometric functions.

## ● TRANSFORMATIONS OF PARENT FUNCTIONS

- **F-IF.4-2** For an exponential, polynomial, trigonometric, or logarithmic function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; end behavior; symmetries; and periodicity.
- **F-BF.3-2** Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $kf(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs, limiting the function types to polynomial, exponential, logarithmic, and trigonometric functions.
- **HS-D.2-10** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-BF.A, F-BF.3, F-IF.3, ACED.1, A-SSE.3, F-IF.B, F-IF.7.
- **HS-Int.3-3** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-LE, A-CED.1, A-SSE.3, F-IF.B, F-IF.7.
- **F-IF.7e-1** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. e) Graph exponential functions, showing intercepts and end behavior.
- **HS-C.9.2** Express reasoning about transformations of functions. Content scope: F-BF.3, which may involve polynomial, exponential, logarithmic, or trigonometric functions. Tasks also may involve even and odd functions.
- **F-IF.7e-2** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. e) Graph logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

## ● MULTIPLE TRANSFORMATIONS OF PARENT FUNCTIONS

- **F-IF.4-2** For an exponential, polynomial, trigonometric, or logarithmic function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; end behavior; symmetries; and periodicity.
- **F-BF.3-2** Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $kf(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs, limiting the function types to polynomial, exponential, logarithmic, and trigonometric functions.
- **HS-D.2-10** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-BF.A, F-BF.3, F-IF.3, ACED.1, A-SSE.3, F-IF.B, F-IF.7.
- **HS-Int.3-3** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-LE, A-CED.1, A-SSE.3, F-IF.B, F-IF.7.
- **HS-C.9.2** Express reasoning about transformations of functions. Content scope: F-BF.3, which may involve polynomial, exponential, logarithmic, or trigonometric functions. Tasks also may involve even and odd functions.
- **F-IF.7e-1** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. e) Graph exponential functions, showing intercepts and end behavior.
- **F-IF.7e-2** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. e) Graph logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

## 28. STATISTICAL DESIGN AND ANALYSIS 1

### ● ANALYZING STATISTICAL SAMPLES

- **HS-C.17.3** Make inferences and justify conclusions from data. Content scope: S-IC.3
- **HS-D.2-13** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in S-ID and S-IC.
- **HS-C.17.2** Make inferences and justify conclusions from data. Content scope: S-IC
- **S-IC.Int.1** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in S-IC.
- **HS-C.17.4** Make inferences and justify conclusions from data. Content scope: S-IC.5

- **S-IC.3-1** Recognize the purposes of and differences among sample surveys, experiments, and observational studies.
- **S-IC.2** Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

## 29. STATISTICAL DESIGN AND ANALYSIS 2

### ● EXPERIMENTAL AND OBSERVATIONAL DESIGN

- **HS-C.17.3** Make inferences and justify conclusions from data. Content scope: S-IC.3
- **HS-D.2-13** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in S-ID and S-IC.
- **HS-C.17.2** Make inferences and justify conclusions from data. Content scope: S-IC
- **S-IC.Int.1** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in S-IC.
- **S-IC.3-1** Recognize the purposes of and differences among sample surveys, experiments, and observational studies.

### ● CONCLUSIONS IN DATA

- **S-IC.Int.1** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in S-IC.
- **HS-D.3-5** Decisions from data: Identify relevant data in a data source, analyze it, and draw reasonable conclusions from it. Content scope: Knowledge and skills articulated in Algebra II.
- **HS-C.17.5** Make inferences and justify conclusions from data. Content scope: S-IC.6
- **HS-C.17.4** Make inferences and justify conclusions from data. Content scope: S-IC.5

## 30. STATISTICS

### ● SCATTERPLOTS AND MODELING

- **HS-D.2-13** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in S-ID and S-IC.
- **S-ID.6a-1** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in S-ID.6a, excluding normal distributions and limiting function fitting to exponential functions.
- **S-ID.6a-2** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in S-ID.6a, excluding normal distributions and limiting function fitting to trigonometric functions.

### ● NORMAL DISTRIBUTION

- **S-ID.4** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
- **HS-D.2-13** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in S-ID and S-IC.

## 31. PROBABILITY

### ● INTRODUCTION TO PROBABILITY

- **S-CP.Int.1** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in S-CP.

### ● CONDITIONAL PROBABILITY

- **S-CP.Int.1** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in S-CP.

## 32. TEST-TAKING STRATEGIES

- **STUDY HABITS**
- **BEING PREPARED AND GETTING STARTED**
- **WORDING IN TEST QUESTIONS**
- **WORDING IN ANSWER CHOICES**
- **QUESTIONS WITH PASSAGES AND VISUAL DATA**
- **ESSAY AND SHORT ANSWER QUESTIONS**
- **WORD PROBLEMS**