

Tennessee Tutorials are designed specifically for the Tennessee Academic Standards to prepare students for the Tennessee Comprehensive Assessment Program (TCAP) and the TNReady assessments.

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

## 1. REAL NUMBER SYSTEM

### • LAWS OF EXPONENTS

- **A2.A.SSE.A.1** Use the structure of an expression to identify ways to rewrite it.
- **A2.A.APR.C.4** Rewrite rational expressions in different forms.
- **A2.A.REI.A.1** Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- **A2.N.RN.A.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
- **A2.N.RN.A.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.

### • MONITORING PRECISION AND ACCURACY

- **A2.N.Q.A.1** Identify, interpret, and justify appropriate quantities for the purpose of descriptive modeling.

## 2. EQUATIONS AND INEQUALITIES

### • LITERAL EQUATIONS

- **A2.A.CED.A.1** Create equations and inequalities in one variable and use them to solve problems.
- **A2.A.CED.A.2** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

### • FORMULATING AND SOLVING EQUATIONS FROM WORD PROBLEMS

- **A2.A.CED.A.1** Create equations and inequalities in one variable and use them to solve problems.
- **A2.F.BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **A2.F.LE.A.1** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs.
- **A2.F.IF.A.1** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

### • FORMULATING AND SOLVING INEQUALITIES FROM WORD PROBLEMS

- **A2.A.CED.A.1** Create equations and inequalities in one variable and use them to solve problems.

### 3. SEQUENCES

#### • SEQUENCES

- **A2.F.BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **A2.F.BF.A.2** Know and write arithmetic and geometric sequences with an explicit formula and use them to model situations.
- **A2.F.LE.A.1** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs.

#### • ARITHMETIC AND GEOMETRIC SEQUENCES

- **A2.F.BF.A.2** Know and write arithmetic and geometric sequences with an explicit formula and use them to model situations.
- **A2.F.BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **A2.F.LE.A.1** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs.

#### • SUMS OF GEOMETRIC SEQUENCES

- **A2.A.SSE.B.3** Recognize a finite geometric series (when the common ratio is not 1), and know and use the sum formula to solve problems in context.
- **A2.F.BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **A2.F.BF.A.2** Know and write arithmetic and geometric sequences with an explicit formula and use them to model situations.

### 4. LINEAR FUNCTIONS AND EQUATIONS

#### • GRAPHING AND ANALYZING LINEAR FUNCTIONS

- **A2.F.LE.A.1** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs.
- **A2.F.IF.A.1** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

#### • GRAPHING AND MANIPULATING $Y = MX + B$

- **A2.F.IF.A.2** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- **A2.F.LE.A.1** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs.
- **A2.F.IF.A.1** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **A2.F.LE.B.3** Interpret the parameters in a linear or exponential function in terms of a context.

### 5. EXPONENTIAL FUNCTIONS

#### • EXPONENTIAL FUNCTIONS

- **A2.A.SSE.B.2.a** Use the properties of exponents to rewrite expressions for exponential functions.
- **A2.F.IF.B.4.a** Know and use the properties of exponents to interpret expressions for exponential functions.
- **A2.F.IF.A.2** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- **A2.F.LE.A.1** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs.
- **A2.F.IF.A.1** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **A2.F.IF.B.3.c** Graph exponential and logarithmic functions, showing intercepts and end behavior.
- **A2.A.CED.A.1** Create equations and inequalities in one variable and use them to solve problems.
- **A2.F.BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.

- **A2.F.LE.B.3** Interpret the parameters in a linear or exponential function in terms of a context.

- **EXPONENTIAL GROWTH AND DECAY**

- **A2.F.IF.B.4.a** Know and use the properties of exponents to interpret expressions for exponential functions.
- **A2.F.LE.A.1** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs.
- **A2.F.LE.B.3** Interpret the parameters in a linear or exponential function in terms of a context.

## 6. EXPONENTIAL EQUATIONS AND INEQUALITIES

- **SOLVING EXPONENTIAL EQUATIONS**

- **A2.A.SSE.A.1** Use the structure of an expression to identify ways to rewrite it.
- **A2.A.SSE.B.2.a** Use the properties of exponents to rewrite expressions for exponential functions.
- **A2.F.IF.B.4.a** Know and use the properties of exponents to interpret expressions for exponential functions.
- **A2.F.IF.A.1** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **A2.F.LE.A.1** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs.
- **A2.F.IF.B.3.c** Graph exponential and logarithmic functions, showing intercepts and end behavior.
- **A2.F.LE.A.2** For exponential models, express as a logarithm the solution to  $abct = d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.

- **SOLVING EXPONENTIAL INEQUALITIES**

- **A2.F.LE.A.1** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs.
- **A2.A.CED.A.1** Create equations and inequalities in one variable and use them to solve problems.

## 7. LOGARITHMIC EXPRESSIONS, EQUATIONS, AND FUNCTIONS

- **LOGARITHMIC FUNCTIONS**

- **A2.F.IF.A.1** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **A2.F.BF.B.4.a** Find the inverse of a function when the given function is one-to-one.
- **A2.F.LE.A.2** For exponential models, express as a logarithm the solution to  $abct = d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.
- **A2.F.IF.B.3.c** Graph exponential and logarithmic functions, showing intercepts and end behavior.

- **EVALUATING LOGARITHMIC EXPRESSIONS**

- **A2.F.LE.A.2** For exponential models, express as a logarithm the solution to  $abct = d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.

- **SOLVING LOGARITHMIC EQUATIONS**

- **A2.F.IF.A.1** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **A2.F.BF.B.4.a** Find the inverse of a function when the given function is one-to-one.
- **A2.F.LE.A.2** For exponential models, express as a logarithm the solution to  $abct = d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.
- **A2.A.REI.A.1** Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

## 8. ARITHMETIC WITH POLYNOMIALS 1

- **POLYNOMIAL BASICS**

- **A2.A.SSE.A.1** Use the structure of an expression to identify ways to rewrite it.

- **ADDITION AND SUBTRACTION OF POLYNOMIALS**

- **A2.A.SSE.A.1** Use the structure of an expression to identify ways to rewrite it.

## 9. ARITHMETIC WITH POLYNOMIALS 2

- **MULTIPLICATION OF POLYNOMIALS**

- **A2.A.SSE.A.1** Use the structure of an expression to identify ways to rewrite it.

- **DIVISION OF POLYNOMIALS**

- **A2.A.SSE.A.1** Use the structure of an expression to identify ways to rewrite it.
- **A2.A.APR.C.4** Rewrite rational expressions in different forms.

## 10. REPRESENTATIONS OF QUADRATIC FUNCTIONS

- **ANALYZING GRAPHS OF QUADRATIC FUNCTIONS**

- **A2.A.SSE.A.1** Use the structure of an expression to identify ways to rewrite it.
- **A2.F.IF.B.5** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- **A2.F.IF.A.1** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **A2.A.REI.B.3.a** Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ .
- **A2.A.APR.A.2** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

- **MULTIPLE REPRESENTATIONS OF FUNCTIONS**

- **A2.F.IF.A.1** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **A2.F.IF.B.5** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- **A2.F.LE.A.1** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs.

## 11. SOLVING QUADRATIC EQUATIONS 1

- **SOLVING QUADRATIC EQUATIONS BY FACTORING**

- **A2.A.REI.B.3.a** Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ .
- **A2.A.APR.A.2** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- **A2.A.APR.B.3** Know and use polynomial identities to describe numerical relationships.
- **A2.F.BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.

- **QUADRATIC FORMULA**

- **A2.A.REI.B.3.a** Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ .
- **A2.N.CN.B.3** Solve quadratic equations with real coefficients that have complex solutions.

- **A2.F.IF.A.1** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **A2.F.BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.

## 12. SOLVING QUADRATIC EQUATIONS 2

### ● COMPLETING THE SQUARE

- **A2.A.REI.B.3.a** Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ .
- **A2.A.SSE.A.1** Use the structure of an expression to identify ways to rewrite it.

### ● COMPLEX NUMBERS AND QUADRATIC FUNCTIONS

- **A2.N.CN.B.3** Solve quadratic equations with real coefficients that have complex solutions.
- **A2.A.REI.B.3.a** Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ .
- **A2.N.CN.A.1** Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.
- **A2.N.CN.A.2** Know and use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

## 13. FACTORING POLYNOMIALS

### ● FACTORING SPECIAL CASES

- **A2.A.SSE.A.1** Use the structure of an expression to identify ways to rewrite it.
- **A2.A.APR.A.2** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- **A2.A.APR.B.3** Know and use polynomial identities to describe numerical relationships.

### ● FACTORING CUBIC POLYNOMIALS

- **A2.A.SSE.A.1** Use the structure of an expression to identify ways to rewrite it.
- **A2.A.APR.B.3** Know and use polynomial identities to describe numerical relationships.
- **A2.A.APR.A.2** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

## 14. FACTORING POLYNOMIALS AND THE FACTOR THEOREM

### ● FACTORING HIGHER-ORDER POLYNOMIALS

- **A2.A.SSE.A.1** Use the structure of an expression to identify ways to rewrite it.
- **A2.A.APR.A.2** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- **A2.A.APR.B.3** Know and use polynomial identities to describe numerical relationships.

### ● FACTOR THEOREM AND REMAINDER THEOREM

- **A2.A.APR.A.1** Know and apply the Remainder Theorem: For a polynomial  $p(x)$  and a number  $a$ , the remainder on division by  $x - a$  is  $p(a)$ , so  $p(a) = 0$  if and only if  $(x - a)$  is a factor of  $p(x)$ .
- **A2.A.APR.C.4** Rewrite rational expressions in different forms.

## 15. POLYNOMIAL FUNCTIONS AND COMPLEX NUMBERS

### ● GRAPHS OF POLYNOMIAL FUNCTIONS

- **A2.A.APR.A.2** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a

rough graph of the function defined by the polynomial.

- **A2.F.IF.A.1** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **A2.F.IF.B.3.b** Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior.

- **COMPLEX NUMBERS**

- **A2.N.CN.A.1** Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.
- **A2.N.CN.A.2** Know and use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

## 16. POLYNOMIAL IDENTITIES AND COMPLEX NUMBERS

- **POLYNOMIAL IDENTITIES**

- **A2.A.APR.B.3** Know and use polynomial identities to describe numerical relationships.
- **A2.A.REI.B.3.a** Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ .
- **A2.A.SSE.A.1** Use the structure of an expression to identify ways to rewrite it.

- **POLYNOMIAL IDENTITIES AND COMPLEX NUMBERS**

- **A2.A.SSE.A.1** Use the structure of an expression to identify ways to rewrite it.
- **A2.A.APR.B.3** Know and use polynomial identities to describe numerical relationships.
- **A2.N.CN.A.1** Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.
- **A2.N.CN.B.3** Solve quadratic equations with real coefficients that have complex solutions.
- **A2.A.REI.B.3.a** Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ .

## 17. RADICAL EQUATIONS AND FUNCTIONS

- **ANALYZING GRAPHS OF SQUARE ROOT FUNCTIONS**

- **A2.F.IF.A.1** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **A2.F.BF.B.3** Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- **A2.F.IF.B.3.a** Graph square root, cube root, and piecewise defined functions, including step functions and absolute value functions.
- **A2.F.BF.B.4.a** Find the inverse of a function when the given function is one-to-one.

- **SOLVING SQUARE ROOT EQUATIONS**

- **A2.A.REI.A.2** Solve rational and radical equations in one variable, and identify extraneous solutions when they exist.
- **A2.A.REI.A.1** Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- **A2.F.BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.

## 18. RATIONAL EXPRESSIONS AND EQUATIONS

- **OPERATIONS WITH RATIONAL EXPRESSIONS**

- **A2.A.SSE.A.1** Use the structure of an expression to identify ways to rewrite it.

- **A2.A.APR.C.4** Rewrite rational expressions in different forms.

- **SOLVING RATIONAL EQUATIONS**

- **A2.A.REI.A.1** Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- **A2.A.REI.A.2** Solve rational and radical equations in one variable, and identify extraneous solutions when they exist.

## 19. REPRESENTATIONS OF RATIONAL FUNCTIONS

- **ANALYZING GRAPHS OF RATIONAL FUNCTIONS**

- **A2.F.IF.A.1** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

- **MODELING SITUATIONS WITH RATIONAL FUNCTIONS**

- **A2.A.REI.A.2** Solve rational and radical equations in one variable, and identify extraneous solutions when they exist.
- **A2.F.BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.

## 20. FUNCTIONS

- **INVERSE FUNCTIONS**

- **A2.F.BF.B.4.a** Find the inverse of a function when the given function is one-to-one.

- **LINEAR VERSUS NONLINEAR FUNCTIONS**

- **A2.F.IF.A.2** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- **A2.F.LE.A.1** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs.
- **A2.F.IF.A.1** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

- **ARITHMETIC OPERATIONS ON FUNCTIONS**

- **A2.F.BF.A.1.b** Combine standard function types using arithmetic operations.

## 21. TRIGONOMETRIC FUNCTIONS

- **RADIANS AND THE UNIT CIRCLE**

- **A2.F.TF.A.1.a** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- **A2.F.TF.A.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- **A2.F.TF.A.1.b** Use the unit circle to find  $\sin \theta$ ,  $\cos \theta$ , and  $\tan \theta$  when  $\theta$  is a commonly recognized angle between 0 and  $2\pi$ .
- **A2.F.TF.B.3.a** Given a point on a circle centered at the origin, recognize and use the right triangle ratio definitions of  $\sin \theta$ ,  $\cos \theta$ , and  $\tan \theta$  to evaluate the trigonometric functions.

- **TRIGONOMETRIC FUNCTIONS**

- **A2.F.TF.A.1.b** Use the unit circle to find  $\sin \theta$ ,  $\cos \theta$ , and  $\tan \theta$  when  $\theta$  is a commonly recognized angle between 0 and  $2\pi$ .
- **A2.F.TF.B.3.a** Given a point on a circle centered at the origin, recognize and use the right triangle ratio definitions of  $\sin \theta$ ,  $\cos \theta$ , and  $\tan \theta$  to evaluate the trigonometric functions.
- **A2.F.TF.B.3.b** Given the quadrant of the angle, use the identity  $\sin^2 \theta + \cos^2 \theta = 1$  to find  $\sin \theta$  given  $\cos \theta$ , or vice versa.

## 22. PARENT FUNCTIONS AND TRANSFORMATIONS

## ● PARENT FUNCTIONS

- **A2.F.IF.A.1** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **A2.F.IF.B.3.a** Graph square root, cube root, and piecewise defined functions, including step functions and absolute value functions.
- **A2.F.BF.B.3** Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- **A2.F.IF.B.3.b** Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior.
- **A2.F.LE.A.1** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs.
- **A2.F.IF.B.3.c** Graph exponential and logarithmic functions, showing intercepts and end behavior.

## ● TRANSFORMATIONS OF PARENT FUNCTIONS

- **A2.F.IF.A.1** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **A2.F.IF.B.3.a** Graph square root, cube root, and piecewise defined functions, including step functions and absolute value functions.
- **A2.F.BF.B.3** Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- **A2.F.IF.B.3.b** Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior.
- **A2.F.IF.B.3.c** Graph exponential and logarithmic functions, showing intercepts and end behavior.

## ● MULTIPLE TRANSFORMATIONS OF PARENT FUNCTIONS

- **A2.F.BF.B.3** Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- **A2.F.IF.A.1** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **A2.F.IF.B.3.a** Graph square root, cube root, and piecewise defined functions, including step functions and absolute value functions.
- **A2.F.IF.B.3.b** Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior.
- **A2.F.IF.B.3.c** Graph exponential and logarithmic functions, showing intercepts and end behavior.

## 23. SYSTEMS OF EQUATIONS

### ● SOLVING THREE-VARIABLE SYSTEMS OF LINEAR EQUATIONS

- **A2.A.REI.C.4** Write and solve a system of linear equations in context.

### ● SYSTEMS OF NONLINEAR EQUATIONS

- **A2.A.REI.C.5** Solve a system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
- **A2.A.REI.D.6** Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the approximate solutions using technology.
- **A2.F.LE.A.1** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs.

## 24. STATISTICAL DESIGN AND ANALYSIS

### ● ANALYZING STATISTICAL SAMPLES



- **A2.S.IC.A.2** Use data from a sample survey to estimate a population mean or proportion; use a given margin of error to solve a problem in context.

- **EXPERIMENTAL AND OBSERVATIONAL DESIGN**

- **A2.S.IC.A.1** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

- **NORMAL DISTRIBUTION**

- **A2.S.ID.A.1** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages using the Empirical Rule.
- **A2.S.IC.A.2** Use data from a sample survey to estimate a population mean or proportion; use a given margin of error to solve a problem in context.

## 25. PROBABILITY

- **INTRODUCTION TO PROBABILITY**

- **A2.S.CP.A.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
- **A2.S.CP.A.2** Understand that two events  $A$  and  $B$  are independent if the probability of  $A$  and  $B$  occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
- **A2.S.CP.A.4** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
- **A2.S.CP.B.6** Know and apply the Addition Rule,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , and interpret the answer in terms of the model.

- **GEOMETRIC PROBABILITIES**

- **A2.S.CP.A.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
- **A2.S.CP.B.6** Know and apply the Addition Rule,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , and interpret the answer in terms of the model.

- **CONDITIONAL PROBABILITY**

- **A2.S.CP.A.3** Know and understand the conditional probability of  $A$  given  $B$  as  $P(A \text{ and } B)/P(B)$ , and interpret independence of  $A$  and  $B$  as saying that the conditional probability of  $A$  given  $B$  is the same as the probability of  $A$ , and the conditional probability of  $B$  given  $A$  is the same as the probability of  $B$ .
- **A2.S.CP.A.4** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
- **A2.S.CP.B.5** Find the conditional probability of  $A$  given  $B$  as the fraction of  $B$ 's outcomes that also belong to  $A$  and interpret the answer in terms of the model.
- **A2.S.CP.A.2** Understand that two events  $A$  and  $B$  are independent if the probability of  $A$  and  $B$  occurring together is the product of their probabilities, and use this characterization to determine if they are independent.