Massachusetts Tutorials are designed specifically for the Learning Standards found in the Massachusetts Curriculum Frameworks to prepare students for the MCAS tests.

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

1. INTRODUCTORY GEOMETRIC CONCEPTS

- **MONITORING PRECISION AND ACCURACY**
  - **GEO.N-Q.A.3.a** *Describe the effects of approximate error in measurement and rounding on measurements and on computed values from measurements. Identify significant figures in recorded measures and computed values based on the context given and the precision of the tools used to measure.*

- **POINTS, RAYS, LINE SEGMENTS, LINES, AND FIGURES**
  - **GEO.G-CO.A.1** *Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.*
  - **GEO.G-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

- **PARALLEL AND PERPENDICULAR LINES**
  - **GEO.G-CO.A.1** *Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.*
  - **GEO.G-GPE.B.5** *Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).*

2. LINES AND ANGLES

- **PARALLEL LINES AND ANGLE RELATIONSHIPS**
  - **GEO.G-CO.A.1** *Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.*
  - **GEO.G-CO.C.9** *Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

- **PERPENDICULAR BISECTOR AND ANGLE BISECTOR THEOREMS**
  - **GEO.G-CO.A.1** *Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the*

undefined notions of point, line, distance along a line, and distance around a circular arc.

- ○ **GEO.G-CO.C.9** *Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
- ○ **GEO.G-CO.C.10** *Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 °; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
- ○ **GEO.G-CO.C.11.a** *Prove theorems about polygons. Theorems include the measures of interior and exterior angles. Apply properties of polygons to the solutions of mathematical and contextual problems.*

3. COORDINATE GEOMETRY I

- **SLOPE-INTERCEPT FORM OF A LINEAR EQUATION**
  - ○ **GEO.G-GPE.B.5** *Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).*

- **LENGTH AND THE DISTANCE FORMULA**
  - ○ **GEO.G-GPE.B.6** *Find the point on a directed line segment between two given points that partitions the segment in a given ratio.*
  - ○ **GEO.G-GPE.B.7** *Use coordinates to compute perimeters of polygons and areas of triangles and rectangles (e.g., using the distance formula).*
  - ○ **GEO.G-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

4. COORDINATE GEOMETRY II

- **MIDPOINT FORMULA ON THE COORDINATE PLANE**
  - ○ **GEO.G-GPE.B.6** *Find the point on a directed line segment between two given points that partitions the segment in a given ratio.*
  - ○ **GEO.G-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*
  - ○ **GEO.G-GPE.B.7** *Use coordinates to compute perimeters of polygons and areas of triangles and rectangles (e.g., using the distance formula).*

- **CONJECTURES IN COORDINATE GEOMETRY**
  - ○ **GEO.G-CO.C.10** *Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 °; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
  - ○ **GEO.G-GPE.B.4** *Use coordinates to prove simple geometric theorems algebraically, including the distance formula and its relationship to the Pythagorean Theorem.*

5. PERIMETER AND AREA

- **PERIMETER ON THE COORDINATE PLANE**
  - ○ **GEO.G-CO.C.11.a** *Prove theorems about polygons. Theorems include the measures of interior and exterior angles. Apply properties of polygons to the solutions of mathematical and contextual problems.*
  - ○ **GEO.G-GPE.B.7** *Use coordinates to compute perimeters of polygons and areas of triangles and rectangles (e.g., using the distance formula).*
  - ○ **GEO.G-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*
  - ○ **GEO.G-GPE.B.4** *Use coordinates to prove simple geometric theorems algebraically, including the distance formula and its relationship to the Pythagorean Theorem.*

- **AREA ON THE COORDINATE PLANE**

- ○ **GEO.G-GPE.B.7** *Use coordinates to compute perimeters of polygons and areas of triangles and rectangles (e.g., using the distance formula).*
- ○ **GEO.G-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

6. TRANSFORMATIONS

- **TRANSFORMATIONS ON THE COORDINATE PLANE**
  - ○ **GEO.G-CO.A.2** *Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).*
  - ○ **GEO.G-CO.A.5** *Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.*
  - ○ **GEO.G-CO.B.6** *Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.*
  - ○ **GEO.G-CO.A.3** *Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.*
  - ○ **GEO.G-CO.A.4** *Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.*
  - ○ **GEO.G-CO.C.11.a** *Prove theorems about polygons. Theorems include the measures of interior and exterior angles. Apply properties of polygons to the solutions of mathematical and contextual problems.*
  - ○ **GEO.G-SRT.A.2** *Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.*
  - ○ **GEO.G-SRT.A.1.a** *A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.*

- **DILATIONS, TRANSLATIONS, ROTATIONS, AND REFLECTIONS**
  - ○ **GEO.G-CO.A.5** *Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.*
  - ○ **GEO.G-CO.B.6** *Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.*
  - ○ **GEO.G-CO.C.11.a** *Prove theorems about polygons. Theorems include the measures of interior and exterior angles. Apply properties of polygons to the solutions of mathematical and contextual problems.*
  - ○ **GEO.G-SRT.A.2** *Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.*
  - ○ **GEO.G-SRT.A.1.a** *A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.*
  - ○ **GEO.G-CO.A.2** *Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).*
  - ○ **GEO.G-CO.A.3** *Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.*
  - ○ **GEO.G-CO.A.4** *Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.*

7. CONGRUENCE

- **TRIANGLES AND CONGRUENCE TRANSFORMATIONS**
  - ○ **GEO.G-CO.B.6** *Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.*
  - ○ **GEO.G-CO.B.7** *Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.*

- GEO.G-CO.B.8 *Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.*
- GEO.G-CO.C.10 *Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
- GEO.G-SRT.B.5 *Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.*

- **CONGRUENCE OF OTHER POLYGONS**
  - GEO.G-CO.C.11.a *Prove theorems about polygons. Theorems include the measures of interior and exterior angles. Apply properties of polygons to the solutions of mathematical and contextual problems.*
  - GEO.G-MG.A.1 *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*
  - GEO.G-CO.A.5 *Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.*
  - GEO.G-CO.B.6 *Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.*
  - GEO.G-CO.A.2 *Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).*
  - GEO.G-CO.A.3 *Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.*

8. SIMILARITY

- **TRIANGLES AND SIMILARITY TRANSFORMATIONS**
  - GEO.G-CO.C.10 *Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
  - GEO.G-SRT.A.2 *Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.*
  - GEO.G-SRT.A.3 *Use the properties of similarity transformations to establish the Angle-Angle (AA) criterion for two triangles to be similar.*
  - GEO.G-SRT.B.4 *Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
  - GEO.G-SRT.B.5 *Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.*

- **SIMILARITY OF OTHER POLYGONS**
  - GEO.G-CO.C.11.a *Prove theorems about polygons. Theorems include the measures of interior and exterior angles. Apply properties of polygons to the solutions of mathematical and contextual problems.*
  - GEO.G-SRT.A.2 *Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.*

9. TRIANGLES

- **TRIANGLE ANGLE THEOREMS**
  - GEO.G-CO.C.10 *Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
  - GEO.G-CO.C.11.a *Prove theorems about polygons. Theorems include the measures of interior and exterior angles. Apply properties of polygons to the solutions of mathematical and contextual problems.*
  - GEO.G-C.A.3 *Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a*

quadrilateral and other polygons inscribed in a circle.

- **TRIANGLE BISECTORS**
  - **GEO.G-CO.C.9** *Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
  - **GEO.G-CO.C.10** *Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
  - **GEO.G-CO.C.11.a** *Prove theorems about polygons. Theorems include the measures of interior and exterior angles. Apply properties of polygons to the solutions of mathematical and contextual problems.*
  - **GEO.G-SRT.B.4** *Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
  - **GEO.G-SRT.B.5** *Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.*
  - **GEO.G-CO.D.12** *Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Constructions include: copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*
  - **GEO.G-C.A.3** *Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral and other polygons inscribed in a circle.*

- **MEDIANS AND ALTITUDES OF TRIANGLES**
  - **GEO.G-CO.C.10** *Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
  - **GEO.G-CO.C.11.a** *Prove theorems about polygons. Theorems include the measures of interior and exterior angles. Apply properties of polygons to the solutions of mathematical and contextual problems.*

10. RIGHT TRIANGLES AND TRIGONOMETRIC RATIOS

- **PYTHAGOREAN THEOREM**
  - **GEO.G-SRT.C.8** *Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*
  - **GEO.G-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*
  - **GEO.G-SRT.B.4** *Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
  - **GEO.G-SRT.B.5** *Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.*
  - **GEO.G-CO.C.10** *Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

- **TRIGONOMETRIC RATIOS**
  - **GEO.G-SRT.C.6** *Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.*
  - **GEO.G-SRT.C.8** *Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*
  - **GEO.G-SRT.C.7** *Explain and use the relationship between the sine and cosine of complementary angles.*
  - **GEO.G-SRT.B.5** *Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.*
  - **GEO.G-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

11. TRIGONOMETRY AND CONSTRUCTIONS

- **LAWS OF SINE AND COSINE**
  - **GEO.G-SRT.C.8** *Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*
  - **GEO.G-SRT.D.9** *Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.*
  - **GEO.G-SRT.D.10** *Prove the Laws of Sines and Cosines and use them to solve problems.*
  - **GEO.G-SRT.D.11** *Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).*

- **RADIANS AND THE UNIT CIRCLE**
  - **GEO.G-C.B.5** *Derive, using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.*
  - **GEO.G-SRT.C.8** *Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*

- **CONSTRUCTIONS**
  - **GEO.G-CO.D.12** *Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Constructions include: copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*
  - **GEO.G-CO.D.13** *Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.*
  - **GEO.G-C.A.4** *Construct a tangent line from a point outside a given circle to the circle.*

12. CIRCLES I

- **CIRCLE BASICS**
  - **GEO.G-CO.A.1** *Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.*
  - **GEO.G-C.A.2** *Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

- **CENTRAL ANGLES, INSCRIBED ANGLES, AND CHORDS**
  - **GEO.G-C.A.2** *Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
  - **GEO.G-CO.A.1** *Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.*
  - **GEO.G-CO.C.9** *Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
  - **GEO.G-C.B.5** *Derive, using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.*

13. CIRCLES II

- **SECANTS, ANGLES, AND INTERCEPTED ARCS**
  - **GEO.G-CO.A.1** *Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.*
  - **GEO.G-CO.C.9** *Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
  - **GEO.G-C.A.2** *Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

- **TANGENTS, ANGLES, AND INTERCEPTED ARCS**
  - **GEO.G-CO.A.1** *Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.*
  - **GEO.G-CO.C.9** *Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
  - **GEO.G-C.A.2** *Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

14. PROPERTIES OF CIRCLES

- **CONGRUENT AND SIMILAR CIRCLES**
  - **GEO.G-CO.A.1** *Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.*
  - **GEO.G-C.A.1** *Prove that all circles are similar.*
  - **GEO.G-CO.A.4** *Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.*
  - **GEO.G-CO.B.6** *Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.*
  - **GEO.G-SRT.A.2** *Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.*

- **CIRCUMFERENCE AND ARC LENGTH**
  - **GEO.G-CO.A.1** *Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.*
  - **GEO.G-GMD.A.1** *Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
  - **GEO.G-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

- **AREA OF CIRCLES AND SECTORS**
  - **GEO.G-GMD.A.1** *Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
  - **GEO.G-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*
  - **GEO.G-CO.A.1** *Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.*
  - **GEO.G-CO.A.5** *Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.*

15. CONIC SECTIONS

- **CIRCLES**
  - **GEO.G-CO.A.1** *Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.*
  - **GEO.G-GPE.A.1** *Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.*

- **PARABOLAS**
  - **GEO.G-GMD.B.4** *Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-*

dimensional objects generated by rotations of two-dimensional objects.

- ○ **GEO.G-GPE.A.2** *Derive the equation of a parabola given a focus and directrix.*

16. SURFACE AREA

- **SURFACE AREA AND VOLUME OF SPHERES**

  - ○ **GEO.G-GMD.A.2** *Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.*
  - ○ **GEO.G-GMD.A.3** *Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*
  - ○ **GEO.G-GMD.B.4** *Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.*
  - ○ **GEO.G-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

- **SURFACE AREA OF COMPOSITE SOLIDS**

  - ○ **GEO.G-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

- **SURFACE AREA OF SIMILAR SOLIDS**

  - ○ **GEO.G-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

17. VOLUME

- **RELATING TWO-DIMENSIONAL FIGURES TO THREE-DIMENSIONAL SOLIDS**

  - ○ **GEO.G-GMD.B.4** *Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.*

- **VOLUME OF PRISMS AND PYRAMIDS**

  - ○ **GEO.G-GMD.A.2** *Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.*
  - ○ **GEO.G-GMD.A.1** *Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
  - ○ **GEO.G-GMD.A.3** *Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*
  - ○ **GEO.G-GMD.B.4** *Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.*
  - ○ **GEO.G-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

- **VOLUME OF CYLINDERS AND CONES**

  - ○ **GEO.G-GMD.A.1** *Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
  - ○ **GEO.G-GMD.A.2** *Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.*
  - ○ **GEO.G-GMD.A.3** *Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*
  - ○ **GEO.G-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*
  - ○ **GEO.G-GMD.B.4** *Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.*

18. APPLICATIONS OF VOLUME

- **MODELING SITUATIONS WITH GEOMETRY**

- GEO.G-MG.A.2 *Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*
- GEO.G-MG.A.3 *Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*

- **VOLUME OF COMPOSITE SOLIDS**

  - GEO.G-MG.A.1 *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*
  - GEO.G-GMD.A.2 *Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.*

19. BASIC PROBABILITY CONCEPTS

- **INTRODUCTION TO PROBABILITY**

  - GEO.S-CP.A.2 *Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.*
  - GEO.S-CP.B.8 *Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|), and interpret the answer in terms of the model.*
  - GEO.S-CP.A.5 *Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.*
  - GEO.S-CP.A.1 *Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").*
  - GEO.S-CP.B.7 *Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.*

- **COMBINATIONS AND PERMUTATIONS**

  - GEO.S-CP.B.9 *Use permutations and combinations to compute probabilities of compound events and solve problems.*

20. ADVANCED PROBABILITY CONCEPTS

- **CONDITIONAL PROBABILITY**

  - GEO.S-CP.A.3 *Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.*
  - GEO.S-CP.A.5 *Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.*
  - GEO.S-CP.B.6 *Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.*
  - GEO.S-CP.A.2 *Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.*
  - GEO.S-CP.A.4 *Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.*
  - GEO.S-CP.A.1 *Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").*

- **GEOMETRIC PROBABILITIES**

  - GEO.G-MG.A.3 *Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*
  - GEO.S-CP.A.1 *Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").*
  - GEO.S-CP.B.7 *Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.*

- **ANALYZING DECISIONS IN PROBABILITY**