

Massachusetts Tutorials are designed specifically for the Learning Standards found in the Massachusetts Curriculum Frameworks to prepare students for the MCAS tests.

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

## 1. REAL NUMBER SYSTEM

### • LAWS OF EXPONENTS

- **AI.A-SSE.A.2** Use the structure of an expression to identify ways to rewrite it.
- **AI.A-REI.A.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method.
- **AI.N-RN.A.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
- **AI.N-RN.A.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.

### • OPERATIONS ON RATIONAL AND IRRATIONAL NUMBERS

- **AI.N-RN.B.3** Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

### • MONITORING PRECISION AND ACCURACY

- **AI.N-Q.A.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
- **AI.N-Q.A.2** Define appropriate quantities for the purpose of descriptive modeling.
- **AI.N-Q.A.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

## 2. EQUATIONS AND INEQUALITIES

### • ONE-STEP EQUATIONS AND INEQUALITIES

- **AI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear, quadratic, and exponential functions with integer exponents.)
- **AI.A-REI.A.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method.
- **AI.F-BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.

- **MULTI-STEP EQUATIONS AND INEQUALITIES**

- **AI.A-REI.A.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method.
- **AI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear, quadratic, and exponential functions with integer exponents.)

### 3. APPLICATIONS OF EQUATIONS

- **AXIOMS OF EQUALITY**

- **AI.A-SSE.A.2** Use the structure of an expression to identify ways to rewrite it.
- **AI.A-REI.A.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method.

- **LITERAL EQUATIONS**

- **AI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear, quadratic, and exponential functions with integer exponents.)
- **AI.A-CED.A.4** Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations (Properties of equality).

### 4. WRITING EXPRESSIONS AND EQUATIONS

- **FORMULATING AND SIMPLIFYING ALGEBRAIC EXPRESSIONS**

- **AI.F-BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **AI.A-SSE.A.1.a** Interpret parts of an expression, such as terms, factors, and coefficients.
- **AI.A-SSE.A.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **AI.A-APR.A.1.b** Factor and/or expand polynomial expressions, identify and combine like terms, and apply the Distributive property.
- **AI.A-SSE.A.2** Use the structure of an expression to identify ways to rewrite it.

- **FORMULATING AND SOLVING EQUATIONS FROM WORD PROBLEMS**

- **AI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear, quadratic, and exponential functions with integer exponents.)
- **AI.F-BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **AI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).
- **AI.F-LE.A.1.b** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- **AI.A-CED.A.3** Represent constraints by linear equations or inequalities, and by systems of linear equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- **AI.F-IF.C.9** Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- **AI.F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.
- **AI.A-SSE.A.1.a** Interpret parts of an expression, such as terms, factors, and coefficients.

- **FORMULATING AND SOLVING INEQUALITIES FROM WORD PROBLEMS**

- **AI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear, quadratic, and exponential functions with integer exponents.)
- **AI.A-CED.A.3** Represent constraints by linear equations or inequalities, and by systems of linear equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- **AI.A-SSE.A.1.a** Interpret parts of an expression, such as terms, factors, and coefficients.

## 5. FUNCTIONS

### • FUNCTIONS AND RELATIONS

- **AI.F-IF.A.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- **AI.F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output (range) of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **AI.F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.
- **AI.F-IF.C.9** Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- **AI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear, quadratic, and exponential functions with integer exponents.)
- **AI.F-IF.C.7.b** Graph piecewise-defined functions, including step functions and absolute value functions.
- **AI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).

### • DOMAIN AND RANGE

- **AI.F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output (range) of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **AI.F-IF.B.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

### • EVALUATING FUNCTIONS

- **AI.F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output (range) of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **AI.F-IF.A.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- **AI.F-BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.

## 6. INTRODUCTION TO LINEAR FUNCTIONS

### • SLOPE

- **AI.F-IF.B.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- **AI.F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output (range) of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **AI.F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.
- **AI.F-IF.C.9** Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.

### • GRAPHING AND ANALYZING LINEAR FUNCTIONS

- **AI.F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output (range) of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **AI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).

- **AI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear, quadratic, and exponential functions with integer exponents.)
- **AI.F-IF.C.9** Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- **AI.F-IF.B.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- **AI.F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.
- **AI.F-IF.C.7.a** Graph linear and quadratic functions and show intercepts, maxima, and minima.

## 7. GRAPHS OF LINEAR EQUATIONS AND INEQUALITIES

### ● GRAPHING AND MANIPULATING $Y = MX + B$

- **AI.A-CED.A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **AI.F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output (range) of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **AI.F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.
- **AI.F-IF.B.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- **AI.F-IF.C.7.a** Graph linear and quadratic functions and show intercepts, maxima, and minima.
- **AI.F-IF.C.9** Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- **AI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).
- **AI.S-ID.C.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- **AI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear, quadratic, and exponential functions with integer exponents.)
- **AI.F-LE.A.1.b** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- **AI.F-LE.B.5** Interpret the parameters in a linear or exponential function (of the form  $f(x) = b$  to the  $x$  power  $+ k$ ) in terms of a context.

### ● GRAPHS OF LINEAR INEQUALITIES

- **AI.A-CED.A.3** Represent constraints by linear equations or inequalities, and by systems of linear equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- **AI.A-REI.D.12** Graph the solutions of a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set of a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

## 8. LINEAR EQUATIONS

### ● SLOPE-INTERCEPT FORM OF A LINEAR EQUATION

- **AI.S-ID.C.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- **AI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear, quadratic, and exponential functions with integer exponents.)
- **AI.F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output (range) of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **AI.F-IF.C.7.a** Graph linear and quadratic functions and show intercepts, maxima, and minima.
- **AI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).

- **AI.A-REI.D.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Show that any point on the graph of an equation in two variables is a solution to the equation.
- **AI.F-IF.B.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

#### ● POINT-SLOPE FORM OF A LINEAR EQUATION

- **AI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear, quadratic, and exponential functions with integer exponents.)
- **AI.F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output (range) of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **AI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).
- **AI.A-REI.D.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Show that any point on the graph of an equation in two variables is a solution to the equation.
- **AI.F-IF.C.7.a** Graph linear and quadratic functions and show intercepts, maxima, and minima.

## 9. TWO-VARIABLE LINEAR SYSTEMS

#### ● SOLVING SYSTEMS OF LINEAR EQUATIONS: GUESS AND CHECK

- **AI.A-CED.A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **AI.A-CED.A.3** Represent constraints by linear equations or inequalities, and by systems of linear equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- **AI.F-BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **AI.A-REI.C.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

#### ● SOLVING SYSTEMS OF LINEAR EQUATIONS: GRAPHING

- **AI.A-CED.A.3** Represent constraints by linear equations or inequalities, and by systems of linear equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- **AI.A-REI.C.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
- **AI.A-REI.D.11** Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions and make tables of values. Include cases where  $f(x)$  and/or  $g(x)$  are linear and exponential functions.
- **AI.A-CED.A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

#### ● SOLVING SYSTEMS OF LINEAR INEQUALITIES

- **AI.A-CED.A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **AI.A-CED.A.3** Represent constraints by linear equations or inequalities, and by systems of linear equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- **AI.A-REI.D.12** Graph the solutions of a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set of a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

## 10. SOLVING TWO-VARIABLE LINEAR SYSTEMS ALGEBRAICALLY

#### ● SOLVING SYSTEMS OF LINEAR EQUATIONS: SUBSTITUTION

- **AI.A-CED.A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

- **AI.A-REI.C.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
- **AI.A-CED.A.3** Represent constraints by linear equations or inequalities, and by systems of linear equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- **AI.A-REI.C.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

### ● SOLVING SYSTEMS OF LINEAR EQUATIONS: ELIMINATION

- **AI.A-CED.A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **AI.A-REI.C.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
- **AI.A-CED.A.3** Represent constraints by linear equations or inequalities, and by systems of linear equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- **AI.A-REI.C.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

## II. EXPONENTIAL FUNCTIONS, EQUATIONS, AND INEQUALITIES

### ● EXPONENTIAL FUNCTIONS

- **AI.A-SSE.A.1.a** Interpret parts of an expression, such as terms, factors, and coefficients.
- **AI.A-SSE.A.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **AI.A-SSE.B.3.c** Use the properties of exponents to transform expressions for exponential functions.
- **AI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear, quadratic, and exponential functions with integer exponents.)
- **AI.F-IF.C.8.b** Use the properties of exponents to interpret expressions for exponential functions. Apply to financial situations such as identifying appreciation and depreciation rate for the value of a house or car some time after its initial purchase:  $V_n = P(1+r)^n$ .
- **AI.F-LE.A.1.a** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- **AI.F-IF.B.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- **AI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).
- **AI.F-LE.A.3** Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
- **AI.F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output (range) of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **AI.F-IF.C.9** Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- **AI.F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.
- **AI.F-IF.B.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- **AI.F-IF.C.7.e** Graph exponential functions showing intercepts and end behavior.
- **AI.F-BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **AI.F-LE.A.1.c** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- **AI.F-LE.B.5** Interpret the parameters in a linear or exponential function (of the form  $f(x) = b$  to the  $x$  power  $+ k$ ) in terms of a context.

### ● EXPONENTIAL GROWTH AND DECAY

- **AI.A-SSE.A.1.a** Interpret parts of an expression, such as terms, factors, and coefficients.
- **AI.A-SSE.A.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity.

- **AI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear, quadratic, and exponential functions with integer exponents.)
- **AI.F-IF.C.8.b** Use the properties of exponents to interpret expressions for exponential functions. Apply to financial situations such as identifying appreciation and depreciation rate for the value of a house or car some time after its initial purchase:  $V_n = P(1+r)^n$ .
- **AI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).
- **AI.F-LE.A.1.a** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- **AI.F-LE.A.1.c** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- **AI.F-LE.B.5** Interpret the parameters in a linear or exponential function (of the form  $f(x) = b$  to the  $x$  power  $+k$ ) in terms of a context.
- **AI.A-CED.A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **AI.S-ID.B.6.a** Fit a linear function to the data and use the fitted function to solve problems in the context of the data. Use functions fitted to data or choose a function suggested by the context (emphasize linear and exponential models).
- **AI.F-LE.A.3** Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
- **AI.F-LE.A.1.b** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

#### ● SOLVING EXPONENTIAL INEQUALITIES

- **AI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).
- **AI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear, quadratic, and exponential functions with integer exponents.)
- **AI.A-SSE.A.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **AI.A-CED.A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **AI.F-LE.A.1.c** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

## 12. SEQUENCES

#### ● SEQUENCES

- **AI.F-IF.A.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
- **AI.F-BF.A.2** Write arithmetic and geometric sequences both recursively and with an explicit formula them to model situations, and translate between the two forms.
- **AI.F-BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **AI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).

#### ● ARITHMETIC AND GEOMETRIC SEQUENCES

- **AI.F-BF.A.2** Write arithmetic and geometric sequences both recursively and with an explicit formula them to model situations, and translate between the two forms.
- **AI.F-IF.A.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
- **AI.F-BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **AI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).

## 13. POLYNOMIALS

#### ● POLYNOMIAL BASICS

- **AI.A-SSE.A.1.a** Interpret parts of an expression, such as terms, factors, and coefficients.
- **AI.A-SSE.A.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **AI.A-APR.A.1.b** Factor and/or expand polynomial expressions, identify and combine like terms, and apply the Distributive property.

- **ADDITION AND SUBTRACTION OF POLYNOMIALS**

- **AI.A-APR.A.1.a** Perform operations on polynomial expressions (addition, subtraction, multiplication), and compare the system of polynomials to the system of integers when performing operations.
- **AI.A-APR.A.1.b** Factor and/or expand polynomial expressions, identify and combine like terms, and apply the Distributive property.

- **MULTIPLICATION OF POLYNOMIALS**

- **AI.A-APR.A.1.a** Perform operations on polynomial expressions (addition, subtraction, multiplication), and compare the system of polynomials to the system of integers when performing operations.
- **AI.A-APR.A.1.b** Factor and/or expand polynomial expressions, identify and combine like terms, and apply the Distributive property.

## 14. FACTORING

- **FACTORING QUADRATIC TRINOMIALS**

- **AI.A-SSE.B.3.a** Factor a quadratic expression to reveal the zeros of the function it defines.
- **AI.A-APR.A.1.b** Factor and/or expand polynomial expressions, identify and combine like terms, and apply the Distributive property.
- **AI.A-REI.B.4.b** Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the solutions of a quadratic equation results in non-real solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ .
- **AI.A-SSE.A.1.a** Interpret parts of an expression, such as terms, factors, and coefficients.
- **AI.A-SSE.A.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity.

- **FACTORING SPECIAL CASES**

- **AI.A-SSE.A.1.a** Interpret parts of an expression, such as terms, factors, and coefficients.
- **AI.A-SSE.A.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **AI.A-APR.A.1.b** Factor and/or expand polynomial expressions, identify and combine like terms, and apply the Distributive property.
- **AI.A-SSE.A.2** Use the structure of an expression to identify ways to rewrite it.

- **FACTORING HIGHER-ORDER POLYNOMIALS**

- **AI.A-SSE.A.1.a** Interpret parts of an expression, such as terms, factors, and coefficients.
- **AI.A-SSE.A.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **AI.A-APR.A.1.b** Factor and/or expand polynomial expressions, identify and combine like terms, and apply the Distributive property.
- **AI.A-SSE.A.2** Use the structure of an expression to identify ways to rewrite it.

## 15. GRAPHS OF QUADRATIC FUNCTIONS

- **QUADRATIC FUNCTIONS**

- **AI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear, quadratic, and exponential functions with integer exponents.)
- **AI.F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.
- **AI.F-IF.C.9** Translate among different representations of functions (algebraically, graphically, numerically in tables, or by

verbal descriptions). Compare properties of two functions each represented in a different way.

- **AI.A-CED.A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **AI.A-SSE.A.1.a** Interpret parts of an expression, such as terms, factors, and coefficients.
- **AI.A-SSE.A.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **AI.A-SSE.B.3.b** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- **AI.F-IF.C.8.a** Use the process of factoring and completing the square in a quadratic function to show zeros, maximum/minimum values, and symmetry of the graph, and interpret these in terms of a context.
- **AI.F-BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.

## ● ANALYZING GRAPHS OF QUADRATIC FUNCTIONS

- **AI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear, quadratic, and exponential functions with integer exponents.)
- **AI.F-IF.C.9** Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- **AI.A-SSE.A.2** Use the structure of an expression to identify ways to rewrite it.
- **AI.F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output (range) of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **AI.F-IF.B.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- **AI.A-REI.B.4.b** Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the solutions of a quadratic equation results in non-real solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ .
- **AI.F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.
- **AI.F-IF.C.7.a** Graph linear and quadratic functions and show intercepts, maxima, and minima.
- **AI.F-IF.C.8.a** Use the process of factoring and completing the square in a quadratic function to show zeros, maximum/minimum values, and symmetry of the graph, and interpret these in terms of a context.

## ● REPRESENTATIONS OF QUADRATIC FUNCTIONS

- **AI.A-REI.B.4.a** Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions. Derive the quadratic formula from this form.
- **AI.A-SSE.A.2** Use the structure of an expression to identify ways to rewrite it.
- **AI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear, quadratic, and exponential functions with integer exponents.)
- **AI.F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output (range) of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **AI.F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.
- **AI.F-IF.C.8.a** Use the process of factoring and completing the square in a quadratic function to show zeros, maximum/minimum values, and symmetry of the graph, and interpret these in terms of a context.
- **AI.F-IF.C.9** Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- **AI.F-BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **AI.A-SSE.B.3.a** Factor a quadratic expression to reveal the zeros of the function it defines.
- **AI.A-CED.A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

## 16. SOLVING QUADRATIC FUNCTIONS

## • SOLVING QUADRATIC EQUATIONS BY FACTORING

- **AI.A-SSE.B.3.a** Factor a quadratic expression to reveal the zeros of the function it defines.
- **AI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear, quadratic, and exponential functions with integer exponents.)
- **AI.A-REI.B.4.b** Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the solutions of a quadratic equation results in non-real solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ .
- **AI.F-IF.C.8.a** Use the process of factoring and completing the square in a quadratic function to show zeros, maximum/minimum values, and symmetry of the graph, and interpret these in terms of a context.
- **AI.A-APR.A.1.b** Factor and/or expand polynomial expressions, identify and combine like terms, and apply the Distributive property.
- **AI.F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output (range) of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **AI.F-IF.C.7.a** Graph linear and quadratic functions and show intercepts, maxima, and minima.
- **AI.F-IF.C.9** Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- **AI.F-BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.

## • COMPLETING THE SQUARE

- **AI.A-SSE.B.3.b** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- **AI.A-REI.B.4.a** Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions. Derive the quadratic formula from this form.
- **AI.A-REI.B.4.b** Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the solutions of a quadratic equation results in non-real solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ .
- **AI.F-IF.C.8.a** Use the process of factoring and completing the square in a quadratic function to show zeros, maximum/minimum values, and symmetry of the graph, and interpret these in terms of a context.
- **AI.A-SSE.A.2** Use the structure of an expression to identify ways to rewrite it.
- **AI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear, quadratic, and exponential functions with integer exponents.)
- **AI.F-IF.C.7.a** Graph linear and quadratic functions and show intercepts, maxima, and minima.

## • QUADRATIC FORMULA

- **AI.A-SSE.A.1.a** Interpret parts of an expression, such as terms, factors, and coefficients.
- **AI.A-SSE.A.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **AI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear, quadratic, and exponential functions with integer exponents.)
- **AI.A-REI.B.4.a** Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions. Derive the quadratic formula from this form.
- **AI.A-REI.B.4.b** Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the solutions of a quadratic equation results in non-real solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ .
- **AI.F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output (range) of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **AI.F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.
- **AI.F-BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.

## 17. PARENT FUNCTIONS

### • LINEAR AND EXPONENTIAL PARENT FUNCTIONS

- **AI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear, quadratic, and exponential functions with integer exponents.)
- **AI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).
- **AI.F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output (range) of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **AI.A-REI.D.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Show that any point on the graph of an equation in two variables is a solution to the equation.
- **AI.F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.
- **AI.F-IF.B.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- **AI.F-IF.C.9** Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- **AI.F-IF.C.7.e** Graph exponential functions showing intercepts and end behavior.
- **AI.F-LE.A.1.c** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

### ● QUADRATIC PARENT FUNCTION

- **AI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear, quadratic, and exponential functions with integer exponents.)
- **AI.F-IF.C.7.a** Graph linear and quadratic functions and show intercepts, maxima, and minima.
- **AI.F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output (range) of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **AI.F-IF.B.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

## 18. TRANSFORMATIONS OF PARENT FUNCTIONS

### ● TRANSFORMATIONS OF THE LINEAR AND EXPONENTIAL PARENT FUNCTIONS

- **AI.F-BF.B.3** Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Include linear, quadratic, exponential, and absolute value functions. Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph.

### ● TRANSFORMATIONS OF THE QUADRATIC PARENT FUNCTION

- **AI.F-BF.B.3** Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Include linear, quadratic, exponential, and absolute value functions. Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph.
- **AI.F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output (range) of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **AI.F-IF.B.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

## 19. NONLINEAR FUNCTIONS

### ● LINEAR VERSUS NONLINEAR FUNCTIONS

- **AI.F-IF.B.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- **AI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).
- **AI.F-LE.A.1.a** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- **AI.F-LE.A.1.b** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

- **AI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear, quadratic, and exponential functions with integer exponents.)
- **AI.F-IF.C.9** Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- **AI.F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.

### ● ABSOLUTE VALUE FUNCTIONS

- **AI.F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output (range) of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **AI.F-IF.B.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- **AI.A-REI.B.3.a** Solve linear equations and inequalities in one variable involving absolute value.
- **AI.F-IF.C.7.b** Graph piecewise-defined functions, including step functions and absolute value functions.
- **AI.F-BF.B.3** Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Include linear, quadratic, exponential, and absolute value functions. Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph.

### ● SYSTEMS OF NONLINEAR EQUATIONS

- **AI.A-REI.C.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
- **AI.A-REI.C.7** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
- **AI.A-REI.C.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
- **AI.A-REI.D.11** Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions and make tables of values. Include cases where  $f(x)$  and/or  $g(x)$  are linear and exponential functions.
- **AI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear, quadratic, and exponential functions with integer exponents.)
- **AI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).

## 20. WORKING WITH FUNCTIONS

### ● ARITHMETIC OPERATIONS ON FUNCTIONS

- **AI.F-BF.A.1.b** Combine standard function types using arithmetic operations.

### ● MULTIPLE REPRESENTATIONS OF FUNCTIONS

- **AI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear, quadratic, and exponential functions with integer exponents.)
- **AI.A-CED.A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **AI.F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.
- **AI.F-IF.C.9** Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- **AI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).
- **AI.F-LE.A.1.a** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

- **INVERSE FUNCTIONS**

- **AI.F-BF.B.4.a** Solve an equation of the form  $f(x) = c$  for a linear function  $f$  that has an inverse and write an expression for the inverse.

## 21. STATISTICS

- **DATA ANALYSIS**

- **AI.S-ID.A.1** Represent data with plots on the real number line (dot plots, histograms, and box plots).
- **AI.S-ID.A.2** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
- **AI.S-ID.A.3** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

- **FREQUENCY TABLES**

- **AI.S-ID.B.5** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
- **AI.S-ID.A.3** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

- **NORMAL DISTRIBUTION**

- **AI.S-ID.A.3** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

## 22. TWO-VARIABLE DATA

- **SCATTERPLOTS**

- **AI.S-ID.B.6.b** Informally assess the fit of a function by plotting and analyzing residuals.
- **AI.S-ID.B.6.c** Fit a linear function for a scatter plot that suggests a linear association.
- **AI.S-ID.C.9** Distinguish between correlation and causation.
- **AI.S-ID.C.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- **AI.F-IF.B.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

- **SCATTERPLOTS AND MODELING**

- **AI.S-ID.B.6.b** Informally assess the fit of a function by plotting and analyzing residuals.
- **AI.S-ID.C.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- **AI.S-ID.B.6.a** Fit a linear function to the data and use the fitted function to solve problems in the context of the data. Use functions fitted to data or choose a function suggested by the context (emphasize linear and exponential models).
- **AI.S-ID.B.6.c** Fit a linear function for a scatter plot that suggests a linear association.
- **AI.S-ID.C.8** Compute (using technology) and interpret the correlation coefficient of a linear fit.
- **AI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear, quadratic, and exponential functions with integer exponents.)
- **AI.F-LE.A.1.a** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- **AI.F-LE.A.1.c** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.