Massachusetts Tutorials are designed specifically for the Learning Standards found in the Massachusetts Curriculum Frameworks to prepare students for the MCAS tests.

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

1. EXPRESSIONS, EQUATIONS, AND INEQUALITIES

- **FORMULATING AND SOLVING EQUATIONS FROM WORD PROBLEMS**
  - **MIII.A-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from simple root and rational functions.)*
  - **MIII.F-IF.C.9** *Translate among different representations of functions: (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.*
  - **MIII.F-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
  - **MIII.A-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*

- **FORMULATING AND SOLVING INEQUALITIES FROM WORD PROBLEMS**
  - **MIII.A-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from simple root and rational functions.)*
  - **MIII.A-CED.A.3** *Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*

2. LITERAL EQUATIONS AND GEOMETRIC SEQUENCES

- **LITERAL EQUATIONS**
  - **MIII.A-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from simple root and rational functions.)*

- **SUMS OF GEOMETRIC SEQUENCES**
  - **MIII.A-SSE.B.4** *Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.*

3. LOGARITHMIC FUNCTIONS AND EXPRESSIONS

- **LOGARITHMIC FUNCTIONS**

- **MIII.F-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
- **MIII.F-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
- **MIII.F-IF.C.9** *Translate among different representations of functions: (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.*
- **MIII.F-IF.C.10** *Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.*
- **MIII.F-IF.C.7.e** *Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.*
- **MIII.F-BF.B.4.a** *Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.*
- **MIII.F-LE.A.4** *For exponential models, express as a logarithm the solution to ab to the ct power $= d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.*

- **EVALUATING LOGARITHMIC EXPRESSIONS**
  - **MIII.A-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
  - **MIII.A-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
  - **MIII.F-LE.A.4** *For exponential models, express as a logarithm the solution to ab to the ct power $= d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.*

4. SOLVING EXPONENTIAL AND LOGARITHMIC EQUATIONS

- **SOLVING EXPONENTIAL EQUATIONS**
  - **MIII.F-IF.C.10** *Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.*
  - **MIII.F-IF.C.9** *Translate among different representations of functions: (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.*
  - **MIII.F-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
  - **MIII.F-IF.C.7.e** *Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.*
  - **MIII.F-BF.B.4.a** *Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.*
  - **MIII.F-LE.A.4** *For exponential models, express as a logarithm the solution to ab to the ct power $= d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.*

- **SOLVING LOGARITHMIC EQUATIONS**
  - **MIII.F-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
  - **MIII.F-IF.C.9** *Translate among different representations of functions: (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.*
  - **MIII.F-IF.C.10** *Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.*
  - **MIII.F-BF.B.4.a** *Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.*
  - **MIII.F-LE.A.4** *For exponential models, express as a logarithm the solution to ab to the ct power $= d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.*

5. POLYNOMIALS I

- **POLYNOMIAL BASICS**
  - **MIII.A-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **MIII.A-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
  - **MIII.A-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*

- **ADDITION AND SUBTRACTION OF POLYNOMIALS**
  - **MIII.A-APR.A.1.a** *Perform operations on polynomial expressions (addition, subtraction, multiplication, and division), and compare the system of polynomials to the system of integers when performing operations.*
  - **MIII.A-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*

6. POLYNOMIALS II

- **MULTIPLICATION OF POLYNOMIALS**
  - **MIII.A-APR.A.1.a** *Perform operations on polynomial expressions (addition, subtraction, multiplication, and division), and compare the system of polynomials to the system of integers when performing operations.*
  - **MIII.A-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*

- **DIVISION OF POLYNOMIALS**
  - **MIII.A-APR.A.1.a** *Perform operations on polynomial expressions (addition, subtraction, multiplication, and division), and compare the system of polynomials to the system of integers when performing operations.*
  - **MIII.A-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **MIII.A-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
  - **MIII.A-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
  - **MIII.A-APR.D.6** *Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where (x), $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.*

7. FACTORING POLYNOMIALS AND THE FACTOR THEOREM

- **FACTORING CUBIC POLYNOMIALS**
  - **MIII.A-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **MIII.A-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
  - **MIII.A-APR.C.4** *Prove polynomial identities and use them to describe numerical relationships.*
  - **MIII.A-APR.B.3** *Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.*

- **FACTORING HIGHER-ORDER POLYNOMIALS**
  - **MIII.A-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
  - **MIII.A-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
  - **MIII.A-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **MIII.A-APR.B.3** *Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.*
  - **MIII.A-APR.C.4** *Prove polynomial identities and use them to describe numerical relationships.*

- **FACTOR THEOREM AND REMAINDER THEOREM**
  - **MIII.A-APR.B.2** *Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.*
  - **MIII.A-APR.D.6** *Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where (x), $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.*

8. POLYNOMIALS AND POLYNOMIAL IDENTITIES

- ## GRAPHS OF POLYNOMIAL FUNCTIONS

  - **MIII.A-APR.B.3** *Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.*
  - **MIII.F-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
  - **MIII.F-IF.C.9** *Translate among different representations of functions: (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.*
  - **MIII.F-IF.C.10** *Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.*
  - **MIII.F-IF.C.7.c** *Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.*
  - **MIII.F-BF.B.3** *Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Include simple rational, radical, logarithmic, and trigonometric functions. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

- ## POLYNOMIAL IDENTITIES

  - **MIII.A-APR.C.4** *Prove polynomial identities and use them to describe numerical relationships.*
  - **MIII.A-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **MIII.A-APR.C.5** *Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.*

- ## POLYNOMIAL IDENTITIES AND COMPLEX NUMBERS

  - **MIII.N-CN.C.8** *Extend polynomial identities to the complex numbers.*
  - **MIII.A-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **MIII.A-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
  - **MIII.A-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
  - **MIII.A-APR.C.4** *Prove polynomial identities and use them to describe numerical relationships.*
  - **MIII.N-CN.C.9** *Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.*

9. QUADRATIC FUNCTIONS AND EQUATIONS

- ## REPRESENTATIONS OF QUADRATIC FUNCTIONS

  - **MIII.A-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **MIII.A-CED.A.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
  - **MIII.A-CED.A.3** *Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
  - **MIII.F-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
  - **MIII.F-IF.C.9** *Translate among different representations of functions: (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.*

- ## COMPLETING THE SQUARE

  - **MIII.A-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **MIII.F-IF.C.8.a** *Use the process of factoring in a polynomial function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.*

10. SQUARE ROOT FUNCTIONS AND EQUATIONS

- ## ANALYZING GRAPHS OF SQUARE ROOT FUNCTIONS

  - **MIII.F-BF.B.3** *Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Include simple rational, radical, logarithmic, and trigonometric functions. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

  - **MIII.F-IF.C.7.b** *Graph square root and cube root functions.*

  - **MIII.F-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

  - **MIII.F-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*

  - **MIII.F-BF.B.4.a** *Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.*

- ## SOLVING SQUARE ROOT EQUATIONS

  - **MIII.A-REI.A.2** *Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.*

  - **MIII.A-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from simple root and rational functions.)*

  - **MIII.F-IF.C.9** *Translate among different representations of functions: (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.*

11. RATIONAL EXPRESSIONS AND EQUATIONS

- ## OPERATIONS WITH RATIONAL EXPRESSIONS

  - **MIII.A-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*

  - **MIII.A-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*

  - **MIII.A-APR.D.7** *Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.*

  - **MIII.A-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*

  - **MIII.A-APR.D.6** *Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where (x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.*

- ## SOLVING RATIONAL EQUATIONS

  - **MIII.A-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from simple root and rational functions.)*

  - **MIII.F-IF.C.9** *Translate among different representations of functions: (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.*

  - **MIII.F-IF.C.10** *Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.*

  - **MIII.A-CED.A.3** *Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*

  - **MIII.A-REI.A.2** *Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.*

12. RATIONAL FUNCTIONS

- ## ANALYZING GRAPHS OF RATIONAL FUNCTIONS

  - **MIII.A-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from simple root and rational functions.)*

  - **MIII.F-IF.C.10** *Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.*

- MIII.F-IF.B.4 *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
- MIII.F-IF.C.9 *Translate among different representations of functions: (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.*
- MIII.F-IF.B.5 *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*

- **MODELING SITUATIONS WITH RATIONAL FUNCTIONS**
  - MIII.A-SSE.A.1.a *Interpret parts of an expression, such as terms, factors, and coefficients.*
  - MIII.A-SSE.A.1.b *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
  - MIII.A-CED.A.1 *Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from simple root and rational functions.)*
  - MIII.A-CED.A.2 *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
  - MIII.A-CED.A.3 *Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
  - MIII.A-REI.A.2 *Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.*
  - MIII.F-IF.C.9 *Translate among different representations of functions: (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.*

13. TRIGONOMETRY

- **RADIANS AND THE UNIT CIRCLE**
  - MIII.F-TF.A.1 *Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.*
  - MIII.F-TF.A.2 *Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.*

- **TRIGONOMETRIC FUNCTIONS**
  - MIII.F-IF.C.7.e *Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.*
  - MIII.F-TF.B.5 *Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*
  - MIII.F-TF.C.8 *Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant.*
  - MIII.F-TF.A.2 *Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.*

- **LAWS OF SINE AND COSINE**
  - MIII.G-SRT.D.9 *Derive the formula $A = 1/2\ ab\ \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.*
  - MIII.G-SRT.D.10 *Prove the Laws of Sines and Cosines and use them to solve problems.*
  - MIII.G-SRT.D.11 *Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).*

14. FUNCTIONS

- **DOMAIN AND RANGE**
  - MIII.F-IF.B.5 *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*

- **ARITHMETIC OPERATIONS ON FUNCTIONS**
  - MIII.F-BF.A.1.b *Combine standard function types using arithmetic operations.*

- **MULTIPLE REPRESENTATIONS OF FUNCTIONS**
  - **MIII.A-CED.A.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
  - **MIII.F-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
  - **MIII.F-IF.C.9** *Translate among different representations of functions: (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.*
  - **MIII.F-IF.C.10** *Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.*

15. WORKING WITH FUNCTIONS

- **INVERSE FUNCTIONS**
  - **MIII.F-BF.B.4.a** *Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.*

- **ABSOLUTE VALUE FUNCTIONS**
  - **MIII.F-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*

16. PARENT FUNCTIONS AND TRANSFORMATIONS

- **PARENT FUNCTIONS**
  - **MIII.F-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
  - **MIII.F-IF.C.9** *Translate among different representations of functions: (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.*
  - **MIII.F-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
  - **MIII.F-IF.C.10** *Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.*
  - **MIII.F-IF.C.7.b** *Graph square root and cube root functions.*
  - **MIII.F-IF.C.7.c** *Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.*
  - **MIII.F-IF.C.7.e** *Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.*
  - **MIII.F-BF.B.3** *Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Include simple rational, radical, logarithmic, and trigonometric functions. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

- **TRANSFORMATIONS OF PARENT FUNCTIONS**
  - **MIII.F-BF.B.3** *Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Include simple rational, radical, logarithmic, and trigonometric functions. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
  - **MIII.F-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
  - **MIII.F-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
  - **MIII.F-IF.C.7.b** *Graph square root and cube root functions.*
  - **MIII.F-IF.C.7.c** *Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end*

behavior.

- ○ **MIII.F-IF.C.7.e** *Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.*

- **MULTIPLE TRANSFORMATIONS OF PARENT FUNCTIONS**

  - ○ **MIII.F-BF.B.3** *Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Include simple rational, radical, logarithmic, and trigonometric functions. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

  - ○ **MIII.F-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

  - ○ **MIII.F-IF.C.7.b** *Graph square root and cube root functions.*

  - ○ **MIII.F-IF.C.7.c** *Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.*

  - ○ **MIII.F-IF.C.7.e** *Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.*

17. SYSTEMS OF EQUATIONS AND INEQUALITIES

- **SOLVING THREE-VARIABLE SYSTEMS OF LINEAR EQUATIONS**

  - ○ **MIII.A-CED.A.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

  - ○ **MIII.A-CED.A.3** *Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*

- **SYSTEMS OF NONLINEAR EQUATIONS**

  - ○ **MIII.A-REI.D.11** *Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are polynomial, rational, and logarithmic functions.*

  - ○ **MIII.A-CED.A.3** *Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*

- **SOLVING SYSTEMS OF LINEAR INEQUALITIES**

  - ○ **MIII.A-CED.A.3** *Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*

18. SURFACE AREA

- **SURFACE AREA AND VOLUME OF SPHERES**

  - ○ **MIII.G-GMD.B.4** *Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.*

  - ○ **MIII.G-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

- **SURFACE AREA OF COMPOSITE SOLIDS**

  - ○ **MIII.G-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

- **SURFACE AREA OF SIMILAR SOLIDS**

  - ○ **MIII.G-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

19. THREE-DIMENSIONAL GEOMETRY

- **RELATING TWO-DIMENSIONAL FIGURES TO THREE-DIMENSIONAL SOLIDS**
  - **MIII.G-GMD.B.4** *Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.*

- **MODELING SITUATIONS WITH GEOMETRY**
  - **MIII.G-MG.A.2** *Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*
  - **MIII.G-MG.A.3** *Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*

20. STATISTICAL ANALYSIS AND DESIGN

- **ANALYZING STATISTICAL SAMPLES**
  - **MIII.S-IC.A.1** *Understand statistics as a process for making inferences to be made about population parameters based on a random sample from that population.*
  - **MIII.S-IC.A.2** *Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.*
  - **MIII.S-IC.B.4** *Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.*

- **EXPERIMENTAL AND OBSERVATIONAL DESIGN**
  - **MIII.S-IC.B.3** *Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.*

- **CONCLUSIONS IN DATA**
  - **MIII.S-IC.B.5** *Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.*
  - **MIII.S-IC.B.6** *Evaluate reports based on data.*

21. STATISTICS AND PROBABILITY

- **NORMAL DISTRIBUTION**
  - **MIII.S-ID.A.4** *Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.*
  - **MIII.S-IC.B.4** *Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.*

- **ANALYZING DECISIONS IN PROBABILITY**
  - **MIII.S-MD.B.6** *Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).*
  - **MIII.S-MD.B.7** *Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game and replacing the goalie with an extra skater).*