Massachusetts Tutorials are designed specifically for the Learning Standards found in the Massachusetts Curriculum Frameworks to prepare students for the MCAS tests.

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

1. RATIONAL AND IRRATIONAL NUMBERS

- **OPERATIONS ON RATIONAL AND IRRATIONAL NUMBERS**
  - **MII.N-RN.B.3** *Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.*

- **LAWS OF EXPONENTS**
  - **MII.A-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **MII.N-RN.A.1** *Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.*
  - **MII.N-RN.A.2** *Rewrite expressions involving radicals and rational exponents using the properties of exponents.*

2. EXPRESSIONS AND EQUATIONS I

- **FORMULATING AND SIMPLIFYING ALGEBRAIC EXPRESSIONS**
  - **MII.F-BF.A.1.a** *Determine an explicit expression, a recursive process, or steps for calculation from a context.*
  - **MII.A-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
  - **MII.A-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
  - **MII.A-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **MII.A-APR.A.1.b** *Factor and/or expand polynomial expressions; identify and combine like terms; and apply the Distributive property.*

- **ONE-STEP EQUATIONS AND INEQUALITIES**
  - **MII.A-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.*
  - **MII.F-BF.A.1.a** *Determine an explicit expression, a recursive process, or steps for calculation from a context.*

3. EXPRESSIONS AND EQUATIONS II

- **MULTI-STEP EQUATIONS AND INEQUALITIES**

- **MII.A-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.*

- **LITERAL EQUATIONS**
  - **MII.A-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.*
  - **MII.A-CED.A.4** *Rearrange formulas, including formulas with quadratic terms, to highlight a quantity of interest using the same reasoning as in solving equations (Properties of equality).*

4. FUNCTIONS

- **FUNCTIONS AND RELATIONS**
  - **MII.F-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*
  - **MII.F-IF.C.9** *Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.*
  - **MII.F-IF.C.7.b** *Graph piecewise-defined functions, including step functions and absolute value functions.*

- **DOMAIN AND RANGE**
  - **MII.F-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*

- **MULTIPLE REPRESENTATIONS OF FUNCTIONS**
  - **MII.F-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*
  - **MII.F-IF.C.9** *Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.*
  - **MII.A-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.*
  - **MII.A-CED.A.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

5. POINTS, LINES, AND ANGLES

- **PARALLEL LINES AND ANGLE RELATIONSHIPS**
  - **MII.G-CO.C.9** *Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

- **PERPENDICULAR BISECTOR AND ANGLE BISECTOR THEOREMS**
  - **MII.G-CO.C.9** *Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
  - **MII.G-CO.C.10** *Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
  - **MII.G-CO.C.11.a** *Prove theorems about polygons. Theorems include the measures of interior and exterior angles. Apply properties of polygons to the solutions of mathematical and contextual problems.*

- **CONJECTURES IN COORDINATE GEOMETRY**

- **MII.G-GPE.B.4** *Use coordinates to prove simple geometric theorems algebraically including the distance formula and its relationship to the Pythagorean Theorem.*

6. THE COORDINATE PLANE

- **LENGTH AND THE DISTANCE FORMULA**
  - **MII.G-GPE.B.6** *Find the point on a directed line segment between two given points that partitions the segment in a given ratio.*

- **MIDPOINT FORMULA ON THE COORDINATE PLANE**
  - **MII.G-GPE.B.6** *Find the point on a directed line segment between two given points that partitions the segment in a given ratio.*

7. CONIC SECTIONS

- **CIRCLES**
  - **MII.G-GPE.A.1** *Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.*

- **PARABOLAS**
  - **MII.G-GPE.A.2** *Derive the equation of a parabola given a focus and directrix.*
  - **MII.A-CED.A.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

8. GEOMETRIC TRANSFORMATIONS

- **TRANSFORMATIONS ON THE COORDINATE PLANE**
  - **MII.G-CO.C.11.a** *Prove theorems about polygons. Theorems include the measures of interior and exterior angles. Apply properties of polygons to the solutions of mathematical and contextual problems.*
  - **MII.G-SR.A.2** *Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.*
  - **MII.G-SR.A.1.a** *A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.*
  - **MII.G-SR.A.1.b** *The dilation of a line segment is longer or shorter in the ratio given by the scale factor.*

- **DILATIONS, TRANSLATIONS, ROTATIONS, AND REFLECTIONS**
  - **MII.G-CO.C.11.a** *Prove theorems about polygons. Theorems include the measures of interior and exterior angles. Apply properties of polygons to the solutions of mathematical and contextual problems.*
  - **MII.G-SR.A.2** *Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.*
  - **MII.G-SR.A.1.a** *A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.*
  - **MII.G-SR.A.1.b** *The dilation of a line segment is longer or shorter in the ratio given by the scale factor.*

9. CONGRUENCE AND SIMILARITY

- **TRIANGLES AND CONGRUENCE TRANSFORMATIONS**
  - **MII.G-CO.C.10** *Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
  - **MII.G-SR.B.5** *Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric*

figures.

- **TRIANGLES AND SIMILARITY TRANSFORMATIONS**
  - **MII.G-CO.C.10** *Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
  - **MII.G-SR.A.2** *Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.*
  - **MII.G-SR.B.4** *Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
  - **MII.G-SR.B.5** *Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.*
  - **MII.G-SR.A.3** *Use the properties of similarity transformations to establish the Angle-Angle (AA) criterion for two triangles to be similar.*

- **SIMILARITY OF OTHER POLYGONS**
  - **MII.G-CO.C.11.a** *Prove theorems about polygons. Theorems include the measures of interior and exterior angles. Apply properties of polygons to the solutions of mathematical and contextual problems.*
  - **MII.G-SR.A.2** *Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.*

10. TRIANGLES AND QUADRILATERALS

- **TRIANGLE ANGLE THEOREMS**
  - **MII.G-CO.C.10** *Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
  - **MII.G-CO.C.11.a** *Prove theorems about polygons. Theorems include the measures of interior and exterior angles. Apply properties of polygons to the solutions of mathematical and contextual problems.*
  - **MII.G-C.A.3** *Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral and other polygons inscribed in a circle.*

- **TRIANGLE BISECTORS**
  - **MII.G-CO.C.9** *Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
  - **MII.G-CO.C.10** *Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
  - **MII.G-CO.C.11.a** *Prove theorems about polygons. Theorems include the measures of interior and exterior angles. Apply properties of polygons to the solutions of mathematical and contextual problems.*
  - **MII.G-SR.B.4** *Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
  - **MII.G-SR.B.5** *Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.*
  - **MII.G-C.A.3** *Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral and other polygons inscribed in a circle.*

- **MEDIANS AND ALTITUDES OF TRIANGLES**
  - **MII.G-CO.C.10** *Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

- **MII.G-CO.C.11.a** *Prove theorems about polygons. Theorems include the measures of interior and exterior angles. Apply properties of polygons to the solutions of mathematical and contextual problems.*

11. RIGHT TRIANGLES AND TRIGONOMETRIC RATIOS

- **PYTHAGOREAN THEOREM**
  - **MII.G-SR.C.8** *Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*
  - **MII.G-SR.B.4** *Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
  - **MII.G-SR.B.5** *Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.*
  - **MII.G-CO.C.10** *Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

- **TRIGONOMETRIC RATIOS**
  - **MII.G-SR.C.6** *Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.*
  - **MII.G-SR.C.8** *Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*
  - **MII.G-SR.C.7** *Explain and use the relationship between the sine and cosine of complementary angles.*
  - **MII.G-SR.B.5** *Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.*

12. TRIGONOMETRY AND CONSTRUCTIONS

- **RADIANS AND THE UNIT CIRCLE**
  - **MII.G-C.B.5** *Derive, using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.*
  - **MII.G-SR.C.8** *Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*

- **TRIGONOMETRIC FUNCTIONS**

- **CONSTRUCTIONS**
  - **MII.G-C.A.4** *Construct a tangent line from a point outside a given circle to the circle.*

13. CIRCLES I

- **CIRCLE BASICS**
  - **MII.G-C.A.2** *Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

- **CENTRAL ANGLES, INSCRIBED ANGLES, AND CHORDS**
  - **MII.G-C.A.2** *Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
  - **MII.G-CO.C.9** *Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
  - **MII.G-C.B.5** *Derive, using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.*

14. CIRCLES II

- **SECANTS, ANGLES, AND INTERCEPTED ARCS**
  - **MII.G-CO.C.9** *Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
  - **MII.G-C.A.2** *Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

- **TANGENTS, ANGLES, AND INTERCEPTED ARCS**
  - **MII.G-CO.C.9** *Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
  - **MII.G-C.A.2** *Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

15. ADVANCED CIRCLE PROPERTIES

- **CONGRUENT AND SIMILAR CIRCLES**
  - **MII.G-C.A.1** *Prove that all circles are similar.*
  - **MII.G-SR.A.2** *Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.*

- **CIRCUMFERENCE AND ARC LENGTH**
  - **MII.G-GMD.A.1** *Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.*

- **AREA OF CIRCLES AND SECTORS**
  - **MII.G-GMD.A.1** *Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
  - **MII.G-C.B.5** *Derive, using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.*

16. EXPONENTIAL FUNCTIONS, EQUATIONS, AND INEQUALITIES

- **EXPONENTIAL FUNCTIONS**
  - **MII.A-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
  - **MII.A-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
  - **MII.A-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.*
  - **MII.F-IF.C.8.b** *Use the properties of exponents to interpret expressions for exponential functions. Apply to financial situations such as Identifying appreciation/depreciation rate for the value of a house or car some time after its initial purchase: V subscript n = $P(1+r)^n$.*
  - **MII.F-IF.B.6** *Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
  - **MII.F-LE.A.3** *Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.*
  - **MII.F-IF.C.9** *Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.*
  - **MII.A-SSE.B.3.c** *Use the properties of exponents to transform expressions for exponential functions.*
  - **MII.F-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features*

include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.

- **MII.F-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
- **MII.F-BF.A.1.a** *Determine an explicit expression, a recursive process, or steps for calculation from a context.*

- **EXPONENTIAL GROWTH AND DECAY**
  - **MII.A-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
  - **MII.A-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
  - **MII.A-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.*
  - **MII.F-IF.C.8.b** *Use the properties of exponents to interpret expressions for exponential functions. Apply to financial situations such as Identifying appreciation/depreciation rate for the value of a house or car some time after its initial purchase: V subscript n = P(1+r)^n.*
  - **MII.A-CED.A.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
  - **MII.F-LE.A.3** *Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.*

- **SOLVING EXPONENTIAL INEQUALITIES**
  - **MII.A-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
  - **MII.A-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.*
  - **MII.A-CED.A.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

17. POLYNOMIALS I

- **POLYNOMIAL BASICS**
  - **MII.A-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
  - **MII.A-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
  - **MII.A-APR.A.1.b** *Factor and/or expand polynomial expressions; identify and combine like terms; and apply the Distributive property.*

- **ADDITION AND SUBTRACTION OF POLYNOMIALS**
  - **MII.A-APR.A.1.a** *Perform operations on polynomial expressions (addition, subtraction, multiplication), and compare the system of polynomials to the system of integers when performing operations.*
  - **MII.A-APR.A.1.b** *Factor and/or expand polynomial expressions; identify and combine like terms; and apply the Distributive property.*

18. POLYNOMIALS II

- **MULTIPLICATION OF POLYNOMIALS**
  - **MII.A-APR.A.1.a** *Perform operations on polynomial expressions (addition, subtraction, multiplication), and compare the system of polynomials to the system of integers when performing operations.*
  - **MII.A-APR.A.1.b** *Factor and/or expand polynomial expressions; identify and combine like terms; and apply the Distributive property.*

- **ARITHMETIC OPERATIONS ON FUNCTIONS**
  - **MII.F-BF.A.1.b** *Combine standard function types using arithmetic operations.*

19. FACTORING

- **FACTORING QUADRATIC TRINOMIALS**

- **MII.A-SSE.B.3.a** *Factor a quadratic expression to reveal the zeros of the function it defines.*
- **MII.A-APR.A.1.b** *Factor and/or expand polynomial expressions; identify and combine like terms; and apply the Distributive property.*
- **MII.A-REI.B.4.b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.*
- **MII.A-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
- **MII.A-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*

FACTORING SPECIAL CASES

- **MII.A-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
- **MII.A-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
- **MII.A-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
- **MII.A-APR.A.1.b** *Factor and/or expand polynomial expressions; identify and combine like terms; and apply the Distributive property.*

20. COMPLEX NUMBERS

COMPLEX NUMBERS

- **MII.N-CN.A.1** *Know there is a complex number i such that $i^2 = –1$, and every complex number has the form $a + bi$ with a and b real.*
- **MII.N-CN.A.2** *Use the relation $i^2 = –1$ and the Commutative, Associative, and Distributive properties to add, subtract, and multiply complex numbers.*

POLYNOMIAL IDENTITIES AND COMPLEX NUMBERS

- **MII.A-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
- **MII.A-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
- **MII.A-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
- **MII.A-APR.A.1.b** *Factor and/or expand polynomial expressions; identify and combine like terms; and apply the Distributive property.*
- **MII.N-CN.A.1** *Know there is a complex number i such that $i^2 = –1$, and every complex number has the form $a + bi$ with a and b real.*
- **MII.N-CN.C.7** *Solve quadratic equations with real coefficients that have complex solutions.*
- **MII.A-REI.B.4.b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.*

21. REPRESENTATIONS OF QUADRATIC FUNCTIONS

QUADRATIC FUNCTIONS

- **MII.A-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.*
- **MII.F-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*
- **MII.F-IF.C.9** *Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.*
- **MII.A-CED.A.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
- **MII.A-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
- **MII.A-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
- **MII.A-SSE.B.3.b** *Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.*
- **MII.F-IF.C.7.a** *Graph quadratic functions and show intercepts, maxima, and minima.*

- **MII.F-IF.C.8.a** *Use the process of factoring and completing the square in a quadratic function to show zeros, minimum/maximum values, and symmetry of the graph and interpret these in terms of a context.*
- **MII.F-BF.A.1.a** *Determine an explicit expression, a recursive process, or steps for calculation from a context.*

- **ANALYZING GRAPHS OF QUADRATIC FUNCTIONS**
  - **MII.A-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **MII.A-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.*
  - **MII.F-IF.C.9** *Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.*
  - **MII.F-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
  - **MII.F-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*
  - **MII.F-IF.C.7.a** *Graph quadratic functions and show intercepts, maxima, and minima.*
  - **MII.F-IF.C.8.a** *Use the process of factoring and completing the square in a quadratic function to show zeros, minimum/maximum values, and symmetry of the graph and interpret these in terms of a context.*

- **REPRESENTATIONS OF QUADRATIC FUNCTIONS**
  - **MII.A-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **MII.A-REI.B.4.a** *Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.*
  - **MII.A-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.*
  - **MII.F-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*
  - **MII.F-IF.C.9** *Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.*
  - **MII.F-BF.A.1.a** *Determine an explicit expression, a recursive process, or steps for calculation from a context.*
  - **MII.A-SSE.B.3.c** *Use the properties of exponents to transform expressions for exponential functions.*
  - **MII.A-CED.A.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
  - **MII.F-IF.C.8.a** *Use the process of factoring and completing the square in a quadratic function to show zeros, minimum/maximum values, and symmetry of the graph and interpret these in terms of a context.*

22. QUADRATIC PARENT FUNCTIONS AND TRANSFORMATIONS

- **QUADRATIC PARENT FUNCTION**
  - **MII.A-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.*
  - **MII.F-IF.C.7.a** *Graph quadratic functions and show intercepts, maxima, and minima.*
  - **MII.F-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*

- **TRANSFORMATIONS OF THE QUADRATIC PARENT FUNCTION**
  - **MII.F-BF.B.3** *Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Include exponential, quadratic, and absolute value functions. Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph.*
  - **MII.F-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*

23. SOLVING QUADRATIC EQUATIONS

- **SOLVING QUADRATIC EQUATIONS BY FACTORING**
  - **MII.A-SSE.B.3.a** *Factor a quadratic expression to reveal the zeros of the function it defines.*
  - **MII.A-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.*
  - **MII.A-REI.B.4.b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.*
  - **MII.F-IF.C.8.a** *Use the process of factoring and completing the square in a quadratic function to show zeros, minimum/maximum values, and symmetry of the graph and interpret these in terms of a context.*
  - **MII.A-APR.A.1.b** *Factor and/or expand polynomial expressions; identify and combine like terms; and apply the Distributive property.*
  - **MII.F-IF.C.7.a** *Graph quadratic functions and show intercepts, maxima, and minima.*
  - **MII.F-IF.C.9** *Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.*
  - **MII.F-BF.A.1.a** *Determine an explicit expression, a recursive process, or steps for calculation from a context.*

- **COMPLETING THE SQUARE**
  - **MII.A-SSE.B.3.b** *Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.*
  - **MII.A-REI.B.4.a** *Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.*
  - **MII.A-REI.B.4.b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.*
  - **MII.F-IF.C.8.a** *Use the process of factoring and completing the square in a quadratic function to show zeros, minimum/maximum values, and symmetry of the graph and interpret these in terms of a context.*
  - **MII.A-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **MII.A-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.*
  - **MII.F-IF.C.7.a** *Graph quadratic functions and show intercepts, maxima, and minima.*

24. QUADRATIC FORMULA AND COMPLEX NUMBERS

- **QUADRATIC FORMULA**
  - **MII.A-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
  - **MII.A-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
  - **MII.A-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.*
  - **MII.A-REI.B.4.a** *Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.*
  - **MII.A-REI.B.4.b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.*
  - **MII.F-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*
  - **MII.F-BF.A.1.a** *Determine an explicit expression, a recursive process, or steps for calculation from a context.*

- **COMPLEX NUMBERS AND QUADRATIC FUNCTIONS**
  - **MII.N-CN.A.2** *Use the relation $i^2 = -1$ and the Commutative, Associative, and Distributive properties to add, subtract, and multiply complex numbers.*
  - **MII.N-CN.C.7** *Solve quadratic equations with real coefficients that have complex solutions.*
  - **MII.A-REI.B.4.b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula*

gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

- **MII.N-CN.A.1** *Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.*
- **MII.F-IF.C.9** *Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.*

25. NONLINEAR FUNCTIONS

INVERSE FUNCTIONS

- **MII.F-BF.B.4.a** *Solve an equation of the form $f(x) = c$ for a linear function f that has an inverse and write an expression for the inverse.*

ABSOLUTE VALUE FUNCTIONS

- **MII.F-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
- **MII.F-IF.C.7.b** *Graph piecewise-defined functions, including step functions and absolute value functions.*
- **MII.F-BF.B.3** *Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Include exponential, quadratic, and absolute value functions. Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph.*

SYSTEMS OF NONLINEAR EQUATIONS

- **MII.A-REI.C.7** *Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.*
- **MII.A-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.*

26. VOLUME

VOLUME OF PRISMS AND PYRAMIDS

- **MII.G-GMD.A.2** *Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.*
- **MII.G-GMD.A.1** *Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
- **MII.G-GMD.A.3** *Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*

VOLUME OF CYLINDERS AND CONES

- **MII.G-GMD.A.1** *Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
- **MII.G-GMD.A.2** *Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.*
- **MII.G-GMD.A.3** *Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*

VOLUME OF COMPOSITE SOLIDS

- **MII.G-GMD.A.3** *Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*
- **MII.G-GMD.A.2** *Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.*

27. BASIC PROBABILITY CONCEPTS

INTRODUCTION TO PROBABILITY

- **MII.S-CP.A.2** *Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.*
- **MII.S-CP.B.8** *Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|\ )$, and interpret the answer in terms of the model.*

- **MII.S-CP.A.5** *Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.*
- **MII.S-CP.B.7** *Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.*

- **COMBINATIONS AND PERMUTATIONS**
  - **MII.S-CP.B.9** *Use permutations and combinations to compute probabilities of compound events and solve problems.*

28. ADVANCED PROBABILITY CONCEPTS

- **CONDITIONAL PROBABILITY**
  - **MII.S-CP.A.3** *Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.*
  - **MII.S-CP.A.5** *Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.*
  - **MII.S-CP.B.6** *Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.*
  - **MII.S-CP.A.2** *Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.*
  - **MII.S-CP.A.4** *Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.*
  - **MII.S-CP.A.1** *Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").*

- **GEOMETRIC PROBABILITIES**
  - **MII.S-CP.A.1** *Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").*
  - **MII.S-CP.B.7** *Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.*

- **ANALYZING DECISIONS IN PROBABILITY**