

Massachusetts Tutorials are designed specifically for the Learning Standards found in the Massachusetts Curriculum Frameworks to prepare students for the MCAS tests.

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

## 1. SOLVING EQUATIONS AND INEQUALITIES

### • ONE-STEP EQUATIONS AND INEQUALITIES

- **MI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and exponential functions with integer exponents.
- **MI.A-REI.A.1** Explain each step in solving a simple linear equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method.
- **MI.F-BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.

### • MULTI-STEP EQUATIONS AND INEQUALITIES

- **MI.A-REI.A.1** Explain each step in solving a simple linear equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method.
- **MI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and exponential functions with integer exponents.

## 2. AXIOMS OF EQUALITY AND LITERAL EQUATIONS

### • AXIOMS OF EQUALITY

- **MI.A-REI.A.1** Explain each step in solving a simple linear equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method.

### • LITERAL EQUATIONS

- **MI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and exponential functions with integer exponents.
- **MI.A-CED.A.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning (Properties of equality) as in solving equations.

## 3. WRITING EQUATIONS AND INEQUALITIES

- **FORMULATING AND SOLVING EQUATIONS FROM WORD PROBLEMS**

- **MI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and exponential functions with integer exponents.
- **MI.F-BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **MI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).
- **MI.F-LE.A.1.b** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- **MI.A-CED.A.3** Represent constraints by linear equations or inequalities, and by systems of linear equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- **MI.F-IF.C.9** Translate among different representations of functions: (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- **MI.F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.
- **MI.A-SSE.A.1.a** Interpret parts of an expression, such as terms, factors, and coefficients.

- **FORMULATING AND SOLVING INEQUALITIES FROM WORD PROBLEMS**

- **MI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and exponential functions with integer exponents.
- **MI.A-CED.A.3** Represent constraints by linear equations or inequalities, and by systems of linear equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- **MI.A-SSE.A.1.a** Interpret parts of an expression, such as terms, factors, and coefficients.

## 4. FUNCTIONS

- **FUNCTIONS AND RELATIONS**

- **MI.F-IF.A.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- **MI.F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **MI.F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.
- **MI.F-IF.C.9** Translate among different representations of functions: (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- **MI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and exponential functions with integer exponents.
- **MI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).

- **DOMAIN AND RANGE**

- **MI.F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **MI.F-IF.B.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

- **EVALUATING FUNCTIONS**

- **MI.F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **MI.F-IF.A.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

- **MI.F-BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.

## 5. GRAPHING LINEAR EQUATIONS AND INEQUALITIES

### ● GRAPHING AND ANALYZING LINEAR FUNCTIONS

- **MI.F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **MI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).
- **MI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and exponential functions with integer exponents.
- **MI.F-IF.C.9** Translate among different representations of functions: (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- **MI.F-IF.B.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- **MI.F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.
- **MI.F-IF.C.7.a** Graph linear functions and show intercepts.

### ● GRAPHING AND MANIPULATING $Y = MX + B$

- **MI.A-CED.A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **MI.F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **MI.F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.
- **MI.F-IF.B.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- **MI.F-IF.C.9** Translate among different representations of functions: (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- **MI.F-IF.C.7.a** Graph linear functions and show intercepts.
- **MI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).
- **MI.S-ID.C.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- **MI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and exponential functions with integer exponents.
- **MI.F-LE.A.1.b** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- **MI.F-LE.B.5** Interpret the parameters in a linear function or exponential function (of the form  $f(x) = b$  to the  $x$  power  $+ k$ ) in terms of a context.

### ● GRAPHS OF LINEAR INEQUALITIES

- **MI.A-REI.D.12** Graph the solutions of a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set of a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
- **MI.A-CED.A.3** Represent constraints by linear equations or inequalities, and by systems of linear equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

## 6. LINEAR EQUATIONS

### ● SLOPE

- **MI.F-IF.B.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- **MI.F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **MI.F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.
- **MI.F-IF.C.9** Translate among different representations of functions: (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- **MI.G-GPE.B.5** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

#### ● SLOPE-INTERCEPT FORM OF A LINEAR EQUATION

- **MI.S-ID.C.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- **MI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and exponential functions with integer exponents.
- **MI.F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **MI.F-IF.C.7.a** Graph linear functions and show intercepts.
- **MI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).
- **MI.A-REI.D.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Show that any point on the graph of an equation in two variables is a solution to the equation.
- **MI.G-GPE.B.5** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
- **MI.F-IF.B.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

#### ● POINT-SLOPE FORM OF A LINEAR EQUATION

- **MI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and exponential functions with integer exponents.
- **MI.F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **MI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).
- **MI.F-IF.C.7.a** Graph linear functions and show intercepts.
- **MI.A-REI.D.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Show that any point on the graph of an equation in two variables is a solution to the equation.
- **MI.G-GPE.B.5** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

## 7. POINTS, LINES, AND ANGLES

#### ● POINTS, RAYS, LINE SEGMENTS, LINES, AND FIGURES

- **MI.G-CO.A.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

#### ● PARALLEL AND PERPENDICULAR LINES

- **MI.G-CO.A.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

- **MI.G-GPE.B.5** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

## 8. COORDINATE GEOMETRY I

### ● LENGTH AND THE DISTANCE FORMULA

- **MI.G-CO.A.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
- **MI.G-GPE.B.7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles (e.g., using the distance formula).

### ● PERIMETER ON THE COORDINATE PLANE

- **MI.G-GPE.B.7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles (e.g., using the distance formula).

## 9. COORDINATE GEOMETRY II

### ● AREA ON THE COORDINATE PLANE

- **MI.G-GPE.B.7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles (e.g., using the distance formula).

### ● CONJECTURES IN COORDINATE GEOMETRY

- **MI.G-CO.A.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

## 10. TRANSFORMATIONS

### ● DILATIONS, TRANSLATIONS, ROTATIONS, AND REFLECTIONS

- **MI.G-CO.A.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
- **MI.G-CO.B.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- **MI.G-CO.A.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
- **MI.G-CO.A.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
- **MI.G-CO.A.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

### ● TRANSFORMATIONS ON THE COORDINATE PLANE

- **MI.G-CO.A.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
- **MI.G-CO.A.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
- **MI.G-CO.B.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- **MI.G-CO.A.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
- **MI.G-CO.A.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

## 11. CONGRUENCE AND CONGRUENCE TRANSFORMATIONS

### • TRIANGLES AND CONGRUENCE TRANSFORMATIONS

- **MI.G-CO.B.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- **MI.G-CO.B.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
- **MI.G-CO.B.8** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

### • CONGRUENCE OF OTHER POLYGONS

- **MI.G-CO.A.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
- **MI.G-CO.B.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- **MI.G-CO.A.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
- **MI.G-CO.A.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

## 12. LINEAR SYSTEMS

### • SOLVING SYSTEMS OF LINEAR EQUATIONS: GUESS AND CHECK

- **MI.A-CED.A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **MI.A-CED.A.3** Represent constraints by linear equations or inequalities, and by systems of linear equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- **MI.F-BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **MI.A-REI.C.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

### • SOLVING SYSTEMS OF LINEAR EQUATIONS: GRAPHING

- **MI.A-CED.A.3** Represent constraints by linear equations or inequalities, and by systems of linear equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- **MI.A-REI.C.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
- **MI.A-REI.D.11** Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions and/or make tables of values. Include cases where  $f(x)$  and/or  $g(x)$  are linear and exponential functions.
- **MI.A-CED.A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

## 13. SOLVING LINEAR SYSTEMS ALGEBRAICALLY

### • SOLVING SYSTEMS OF LINEAR EQUATIONS: SUBSTITUTION

- **MI.A-CED.A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **MI.A-REI.C.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
- **MI.A-CED.A.3** Represent constraints by linear equations or inequalities, and by systems of linear equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- **MI.A-REI.C.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that

equation and a multiple of the other produces a system with the same solutions.

- **SOLVING SYSTEMS OF LINEAR EQUATIONS: ELIMINATION**

- **MI.A-CED.A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **MI.A-REI.C.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
- **MI.A-CED.A.3** Represent constraints by linear equations or inequalities, and by systems of linear equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
- **MI.A-REI.C.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

## 14. WORKING WITH FUNCTIONS

- **LINEAR VERSUS NONLINEAR FUNCTIONS**

- **MI.F-IF.B.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- **MI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).
- **MI.F-LE.A.1.a** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- **MI.F-LE.A.1.b** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- **MI.F-LE.A.1.c** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- **MI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and exponential functions with integer exponents.
- **MI.F-IF.C.9** Translate among different representations of functions: (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- **MI.F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.

- **LINEAR AND EXPONENTIAL PARENT FUNCTIONS**

- **MI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and exponential functions with integer exponents.
- **MI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).
- **MI.A-REI.D.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Show that any point on the graph of an equation in two variables is a solution to the equation.
- **MI.F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **MI.F-IF.B.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- **MI.F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.
- **MI.F-IF.C.9** Translate among different representations of functions: (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- **MI.F-IF.C.7.e** Graph exponential functions, showing intercepts and end behavior.
- **MI.F-LE.A.1.c** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

- **TRANSFORMATIONS OF THE LINEAR AND EXPONENTIAL PARENT FUNCTIONS**

- **MI.F-BF.B.3** Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Include linear and exponential models. (Focus on vertical translations for exponential functions). Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph.
- **MI.G-CO.A.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
- **MI.G-CO.B.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

## 15. EXPONENTIAL FUNCTIONS, EQUATIONS, AND INEQUALITIES

### ● EXPONENTIAL FUNCTIONS

- **MI.A-SSE.A.1.a** Interpret parts of an expression, such as terms, factors, and coefficients.
- **MI.A-SSE.A.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **MI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and exponential functions with integer exponents.
- **MI.F-LE.A.1.a** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- **MI.F-IF.B.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- **MI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).
- **MI.F-LE.A.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly.
- **MI.F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **MI.F-IF.C.9** Translate among different representations of functions: (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.
- **MI.F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.
- **MI.F-IF.B.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- **MI.F-IF.C.7.e** Graph exponential functions, showing intercepts and end behavior.
- **MI.F-BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **MI.F-LE.A.1.c** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- **MI.F-LE.B.5** Interpret the parameters in a linear function or exponential function (of the form  $f(x) = b$  to the  $x$  power  $+ k$ ) in terms of a context.

### ● EXPONENTIAL GROWTH AND DECAY

- **MI.A-SSE.A.1.a** Interpret parts of an expression, such as terms, factors, and coefficients.
- **MI.A-SSE.A.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **MI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and exponential functions with integer exponents.
- **MI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).
- **MI.F-LE.A.1.a** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- **MI.F-LE.A.1.c** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- **MI.F-LE.B.5** Interpret the parameters in a linear function or exponential function (of the form  $f(x) = b$  to the  $x$  power  $+ k$ ) in terms of a context.
- **MI.A-CED.A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

- **MI.F-LE.A.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly.
- **MI.S-ID.B.6.a** Fit a linear function to the data and use the fitted function to solve problems in the context of the data. Use given functions fitted to data or choose a function suggested by the context. Emphasize linear and exponential models.
- **MI.F-LE.A.1.b** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

#### ● SOLVING EXPONENTIAL INEQUALITIES

- **MI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).
- **MI.A-SSE.A.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **MI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and exponential functions with integer exponents.
- **MI.A-CED.A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **MI.F-LE.A.1.c** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

## 16. SEQUENCES

#### ● SEQUENCES

- **MI.F-IF.A.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
- **MI.F-BF.A.2** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
- **MI.F-BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **MI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).

#### ● ARITHMETIC AND GEOMETRIC SEQUENCES

- **MI.F-BF.A.2** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
- **MI.F-IF.A.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
- **MI.F-BF.A.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **MI.F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).

## 17. STATISTICS

#### ● DATA ANALYSIS

- **MI.S-ID.A.1** Represent data with plots on the real number line (dot plots, histograms, and box plots).
- **MI.S-ID.A.2** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
- **MI.S-ID.A.3** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

#### ● FREQUENCY TABLES

- **MI.S-ID.B.5** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
- **MI.S-ID.A.3** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

## 18. TWO-VARIABLE DATA

### • SCATTERPLOTS

- **MI.S-ID.B.6.b** Informally assess the fit of a function by plotting and analyzing residuals.
- **MI.S-ID.B.6.c** Fit a linear function for a scatter plot that suggests a linear association.
- **MI.S-ID.C.9** Distinguish between correlation and causation.
- **MI.F-IF.B.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- **MI.S-ID.C.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

### • SCATTERPLOTS AND MODELING

- **MI.S-ID.B.6.b** Informally assess the fit of a function by plotting and analyzing residuals.
- **MI.S-ID.B.6.a** Fit a linear function to the data and use the fitted function to solve problems in the context of the data. Use given functions fitted to data or choose a function suggested by the context. Emphasize linear and exponential models.
- **MI.S-ID.B.6.c** Fit a linear function for a scatter plot that suggests a linear association.
- **MI.S-ID.C.8** Compute (using technology) and interpret the correlation coefficient of a linear fit.
- **MI.S-ID.C.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- **MI.A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and exponential functions with integer exponents.
- **MI.F-LE.A.1.a** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- **MI.F-LE.A.1.c** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

## 19. TOPICS IN GEOMETRY

### • MONITORING PRECISION AND ACCURACY

- **MI.N-Q.A.1** Use units as a way to understand problems; and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
- **MI.N-Q.A.2** Define appropriate quantities for the purpose of descriptive modeling.
- **MI.N-Q.A.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

### • CONSTRUCTIONS

- **MI.G-CO.D.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
- **MI.G-CO.D.13** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.