

Massachusetts Tutorials are designed specifically for the Learning Standards found in the Massachusetts Curriculum Frameworks to prepare students for the MCAS tests.

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

1. RATE, RATIO, AND PROPORTION

• UNIT RATES

- **7.RP.A.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units.
- **7.RP.A.2.a** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table, or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

• IDENTIFYING PROPORTIONAL RELATIONSHIPS

- **7.RP.A.2.a** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table, or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- **7.RP.A.3** Use proportional relationships to solve multi-step ratio, rate, and percent problems.

• USING PROPORTIONS TO SOLVE PROBLEMS

- **7.RP.A.3** Use proportional relationships to solve multi-step ratio, rate, and percent problems.
- **7.RP.A.2.a** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table, or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

2. PROPORTIONAL REASONING

• ANALYZING PROPORTIONAL RELATIONSHIPS

- **7.RP.A.2.b** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- **7.RP.A.2.d** Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.
- **7.RP.A.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units.

• REPRESENTING PROPORTIONAL RELATIONSHIPS

- **7.RP.A.3** Use proportional relationships to solve multi-step ratio, rate, and percent problems.
- **7.RP.A.2.a** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table, or

graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

- **7.RP.A.2.d** Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.
- **7.RP.A.2.c** Represent proportional relationships by equations.

3. ADDITION AND SUBTRACTION OF RATIONAL NUMBERS

• ADDING RATIONAL NUMBERS

- **7.NS.A.1.a** Describe situations in which opposite quantities combine to make zero.
- **7.NS.A.1.b** Understand $p + q$ as the number located a distance $|q|$ from p , in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
- **7.NS.A.3** Solve real-world and mathematical problems involving the four operations with integers and other rational numbers.
- **7.EE.B.3** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

• SUBTRACTING RATIONAL NUMBERS

- **7.NS.A.1.a** Describe situations in which opposite quantities combine to make zero.
- **7.NS.A.1.b** Understand $p + q$ as the number located a distance $|q|$ from p , in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
- **7.NS.A.1.c** Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
- **7.NS.A.3** Solve real-world and mathematical problems involving the four operations with integers and other rational numbers.
- **7.EE.B.3** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

• USING PROPERTIES TO ADD AND SUBTRACT RATIONAL NUMBERS

- **7.NS.A.3** Solve real-world and mathematical problems involving the four operations with integers and other rational numbers.
- **7.EE.B.3** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.
- **7.NS.A.1.d** Apply properties of operations as strategies to add and subtract rational numbers.

4. MULTIPLICATION AND DIVISION OF RATIONAL NUMBERS

• MULTIPLYING RATIONAL NUMBERS

- **7.NS.A.3** Solve real-world and mathematical problems involving the four operations with integers and other rational numbers.
- **7.NS.A.2.a** Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
- **7.EE.B.3** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

• DIVIDING RATIONAL NUMBERS

- **7.NS.A.2.b** Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational

numbers by describing real-world contexts.

- **7.NS.A.3** Solve real-world and mathematical problems involving the four operations with integers and other rational numbers.
- **7.EE.B.3** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.
- **7.NS.A.2.c** Apply properties of operations as strategies to multiply and divide rational numbers.

● USING PROPERTIES TO MULTIPLY AND DIVIDE RATIONAL NUMBERS

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- **7.NS.A.2.a** Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

5. WORKING WITH RATIONAL NUMBERS

● EXPRESSING RATIONAL NUMBERS IN DECIMAL FORM

- **7.EE.B.3** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.
- **7.NS.A.2.d** Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

● USING OPERATIONS ON RATIONAL NUMBERS TO SOLVE PROBLEMS

- **7.NS.A.3** Solve real-world and mathematical problems involving the four operations with integers and other rational numbers.
- **7.NS.A.1.b** Understand $p + q$ as the number located a distance $|q|$ from p , in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
- **7.NS.A.1.c** Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
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- **7.EE.B.3** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

6. ALGEBRAIC EXPRESSIONS

● SIMPLIFYING AND REWRITING ALGEBRAIC EXPRESSIONS

- **7.EE.A.1** Apply properties of operations to add, subtract, factor, and expand linear expressions with rational coefficients.
- **7.EE.A.2** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

● SOLVING MULTI-STEP PROBLEMS WITH RATIONAL NUMBERS

- **7.NS.A.3** Solve real-world and mathematical problems involving the four operations with integers and other rational numbers.
- **7.EE.B.3** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form

(whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

- **7.NS.A.1.d** Apply properties of operations as strategies to add and subtract rational numbers.
- **7.NS.A.2.c** Apply properties of operations as strategies to multiply and divide rational numbers.
- **7.EE.A.2** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

7. EQUATIONS AND INEQUALITIES

• SOLVING TWO-STEP EQUATIONS

- **7.EE.B.4.a** Solve word problems leading to equations of the form $px + q = r$ and $p(x \div q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

• SOLVING LINEAR INEQUALITIES

- **7.EE.B.4.b** Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p , q , and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

8. DRAWING, CONSTRUCTING, AND EXPLORING GEOMETRIC FIGURES

• SCALE DRAWINGS

- **7.G.A.1** Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

• GEOMETRIC DRAWINGS

- **7.G.A.2** Draw (freehand, with ruler and protractor, and with technology) two-dimensional geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
- **7.G.A.3** Describe the shape of the two-dimensional face of the figure that results from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

• CROSS-SECTIONS OF GEOMETRIC SOLIDS

- **7.G.A.3** Describe the shape of the two-dimensional face of the figure that results from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

9. GEOMETRY IN TWO AND THREE DIMENSIONS

• CIRCLES

- **7.G.B.4.b** Understand and describe the relationships among the radius, diameter, and circumference of a circle.
- **7.G.B.4.c** Understand and describe the relationship among the radius, diameter, and area of a circle.
- **7.G.B.4.d** Know the formulas for the area and circumference of a circle and use them to solve problems.
- **7.G.B.4.a** Know that a circle is a two-dimensional shape created by connecting all of the points equidistant from a fixed point called the center of the circle.
- **7.G.B.4.e** Give an informal derivation of the relationship between the circumference and area of a circle.

• ANGLE RELATIONSHIPS

- **7.G.B.5** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write simple equations and use them to solve for an unknown angle in a figure.

• AREA, VOLUME, AND SURFACE AREA

- **7.G.B.6** Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

10. STATISTICS AND SAMPLING

● POPULATIONS AND SAMPLES

- **7.SP.A.1** Understand that statistics can be used to gain information about a population by examining a sample of the population; Generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
- **7.SP.A.2** Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.
- **7.SP.B.4** Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

● COMPARING DATA SETS VISUALLY

- **7.SP.B.3** Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.
- **7.SP.B.4** Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.
- **7.EE.B.3** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

● USING STATISTICAL MEASURES TO COMPARE DATA SETS

- **7.SP.B.3** Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.
- **7.SP.B.4** Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

11. PROBABILITY 1

● PROBABILITY

- **7.SP.C.5** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
- **7.SP.A.1** Understand that statistics can be used to gain information about a population by examining a sample of the population; Generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
- **7.SP.A.2** Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.

● CALCULATING PROBABILITY

- **7.SP.C.7.a** Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.
- **7.SP.C.8.a** Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
- **7.SP.C.6** Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.
- **7.SP.C.7.b** Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.

12. PROBABILITY 2

● PROBABILITY OF COMPOUND EVENTS

- **7.SP.C.5** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability

around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

- **7.SP.C.8.a** Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
- **7.SP.C.8.b** Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.

- **SIMULATIONS**

- **7.SP.C.8.a** Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
- **7.SP.C.8.c** Design and use a simulation to generate frequencies for compound events.
- **7.SP.C.5** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.