

Massachusetts Tutorials are designed specifically for the Learning Standards found in the Massachusetts Curriculum Frameworks to prepare students for the MCAS tests.

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

1. RATIOS AND RATES

• RATIOS

- **6.RP.A.1** Understand the concept of a ratio including the distinctions between part:part and part:whole and the value of a ratio; part/part and part/whole. Use ratio language to describe a ratio relationship between two quantities.
- **6.RP.A.3.a** Make tables of equivalent ratios relating quantities with whole-number measurements. Find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
- **6.NS.C.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.
- **6.NS.C.6.c** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
- **6.NS.C.7.b** Write, interpret, and explain statements of order for rational numbers in real-world contexts.

• RATES AND UNIT RATES

- **6.RP.A.2** Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship, including the use of units.
- **6.RP.A.3.b** Solve unit rate problems, including those involving unit pricing, and constant speed.

2. PERCENTS AND UNIT CONVERSIONS

• SOLVING PERCENT PROBLEMS

- **6.RP.A.3.c** Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $30/100$ times the quantity); solve problems involving finding the whole, given a part and the percent.

• UNIT CONVERSIONS

- **6.RP.A.3.d** Use ratio reasoning to convert measurement units within and between measurement systems; manipulate and transform units appropriately when multiplying or dividing quantities.

3. DIVISION

- **DIVIDING FRACTIONS**

- **6.NS.A.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.
- **6.NS.C.5** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, and positive/negative electric charge). Use positive and negative numbers (whole numbers, fractions, and decimals) to represent quantities in real-world contexts, explaining the meaning of zero in each situation.

- **SOLVING PROBLEMS BY DIVIDING FRACTIONS**

- **6.NS.A.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

- **DIVIDING MULTI-DIGIT WHOLE NUMBERS**

- **6.NS.B.2** Fluently divide multi-digit numbers using the standard algorithm.

4. NUMBER SENSE

- **DECIMAL OPERATIONS**

- **6.NS.B.3** Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

- **GREATEST COMMON FACTOR AND LEAST COMMON MULTIPLE**

- **6.NS.B.4** Use prime factorization to find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two relatively prime numbers.

5. SIGNED NUMBERS

- **SIGNED NUMBERS**

- **6.NS.C.5** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, and positive/negative electric charge). Use positive and negative numbers (whole numbers, fractions, and decimals) to represent quantities in real-world contexts, explaining the meaning of zero in each situation.
- **6.NS.C.6.a** Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that zero is its own opposite.
- **6.NS.C.6.c** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

- **INEQUALITIES AND COMPARISON**

- **6.NS.C.7.a** Interpret statements of inequality as statements about the relative positions of two numbers on a number line diagram.
- **6.EE.B.8** Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
- **6.NS.C.7.b** Write, interpret, and explain statements of order for rational numbers in real-world contexts.

- **ABSOLUTE VALUE**

- **6.NS.C.6.a** Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that zero is its own opposite.
- **6.NS.C.6.c** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
- **6.NS.C.7.c** Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.
- **6.NS.C.7.d** Distinguish comparisons of absolute value from statements about order.

- **6.NS.C.7.b** Write, interpret, and explain statements of order for rational numbers in real-world contexts.

6. THE COORDINATE PLANE

● PLOTTING POINTS IN THE COORDINATE PLANE

- **6.NS.C.6.a** Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that zero is its own opposite.
- **6.NS.C.6.c** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
- **6.NS.C.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.
- **6.NS.C.6.b** Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
- **6.G.A.3** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

● QUADRANTS AND AXES

- **6.NS.C.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.
- **6.NS.C.6.b** Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
- **6.NS.C.6.c** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

● USING GRAPHS TO SOLVE PROBLEMS

- **6.NS.C.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.
- **6.NS.C.6.c** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
- **6.G.A.3** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

7. EXPONENTS AND EXPRESSIONS

● EXPONENTS

- **6.EE.A.1** Write and evaluate numerical expressions involving whole-number exponents.
- **6.EE.A.2.c** Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

● UNDERSTANDING PARTS OF EXPRESSIONS

- **6.EE.A.2.b** Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, and coefficient); view one or more parts of an expression as a single entity.

● EQUIVALENT EXPRESSIONS

- **6.EE.A.3** Apply the properties of operations to generate equivalent expressions.
- **6.EE.A.4** Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).

- **6.EE.A.2.c** Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

8. WRITING AND EVALUATING EXPRESSIONS

• WRITING EXPRESSIONS

- **6.EE.B.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- **6.EE.C.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.
- **6.EE.A.2.a** Write expressions that record operations with numbers and with letters standing for numbers.

• WRITING EXPRESSIONS TO SOLVE PROBLEMS

- **6.EE.B.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- **6.EE.C.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.
- **6.EE.A.2.a** Write expressions that record operations with numbers and with letters standing for numbers.
- **6.EE.A.2.c** Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

• EVALUATING EXPRESSIONS

- **6.EE.A.2.c** Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

9. BASICS OF EQUATIONS

• INDEPENDENT AND DEPENDENT VARIABLES

- **6.EE.B.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- **6.EE.C.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.
- **6.EE.A.2.a** Write expressions that record operations with numbers and with letters standing for numbers.

• MULTIPLE REPRESENTATIONS: TABLES, GRAPHS, AND EQUATIONS

- **6.EE.C.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

10. SOLVING EQUATIONS AND INEQUALITIES 1

• SOLUTIONS OF EQUATIONS AND INEQUALITIES

- **6.EE.B.5** Understand solving an equation or inequality as a process of answering a question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- **6.EE.B.7** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all nonnegative rational numbers.

- **SOLVING ADDITION EQUATIONS**

- **6.EE.B.5** Understand solving an equation or inequality as a process of answering a question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- **6.EE.B.7** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all nonnegative rational numbers.

11. SOLVING EQUATIONS AND INEQUALITIES 2

- **SOLVING MULTIPLICATION EQUATIONS**

- **6.EE.B.5** Understand solving an equation or inequality as a process of answering a question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- **6.EE.B.7** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all nonnegative rational numbers.

- **SOLVING INEQUALITIES**

- **6.EE.B.5** Understand solving an equation or inequality as a process of answering a question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- **6.EE.B.8** Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

12. GEOMETRY 1

- **COORDINATE GEOMETRY**

- **6.NS.C.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.
- **6.G.A.3** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
- **6.NS.C.6.c** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

- **SOLID FIGURES**

- **6.G.A.4** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface areas of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

13. GEOMETRY 2

- **CIRCLES**

- **6.G.A.1** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

- **VOLUME**

- **6.G.A.2** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = Bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

14. INTRODUCTION TO STATISTICS

● STATISTICAL QUESTIONS AND DATA DISTRIBUTIONS

- **6.SP.A.1** Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.
- **6.SP.A.2** Understand that a set of data collected to answer a statistical question has a distribution, which can be described by its center (median, mean, and/or mode), spread (range, interquartile range), and overall shape.
- **6.SP.A.3** Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
- **6.SP.B.5.c** Giving quantitative measures of center (median, and/or mean) and variability (range and/or interquartile range), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
- **6.SP.B.5.b** Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

● MEASURES OF CENTER AND VARIABILITY

- **6.SP.A.2** Understand that a set of data collected to answer a statistical question has a distribution, which can be described by its center (median, mean, and/or mode), spread (range, interquartile range), and overall shape.
- **6.SP.A.3** Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
- **6.SP.B.5.c** Giving quantitative measures of center (median, and/or mean) and variability (range and/or interquartile range), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

15. DATA COLLECTION

● BOX PLOTS

- **6.SP.A.2** Understand that a set of data collected to answer a statistical question has a distribution, which can be described by its center (median, mean, and/or mode), spread (range, interquartile range), and overall shape.
- **6.SP.B.5.c** Giving quantitative measures of center (median, and/or mean) and variability (range and/or interquartile range), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
- **6.SP.B.5.b** Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

● COLLECTING DATA

- **6.SP.B.5.a** Reporting the number of observations.
- **6.SP.B.5.b** Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

16. SUMMARIZING DATA

● SUMMARIZING DATA USING MEASURES OF CENTER AND VARIABILITY

- **6.SP.A.2** Understand that a set of data collected to answer a statistical question has a distribution, which can be described by its center (median, mean, and/or mode), spread (range, interquartile range), and overall shape.
- **6.SP.B.5.c** Giving quantitative measures of center (median, and/or mean) and variability (range and/or interquartile range), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
- **6.SP.A.3** Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

- **CHOOSING APPROPRIATE MEASURES TO SUMMARIZE DATA SETS**

- **6.SP.A.2** Understand that a set of data collected to answer a statistical question has a distribution, which can be described by its center (median, mean, and/or mode), spread (range, interquartile range), and overall shape.
- **6.SP.A.3** Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
- **6.SP.B.5.c** Giving quantitative measures of center (median, and/or mean) and variability (range and/or interquartile range), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
- **6.SP.B.5.d** Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.