Kentucky Tutorials are designed specifically for the Kentucky Academic Standards to prepare students for the K-PREP, EOC exams, ACT, and ACT Plan.

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

This Tutorial is aligned with the Common Core State Standards for Mathematics.

1. EXPRESSIONS, EQUATIONS, AND INEQUALITIES

- **FORMULATING AND SIMPLIFYING ALGEBRAIC EXPRESSIONS**
  - **HSF-BF.A.1.a** *Determine an explicit expression, a recursive process, or steps for calculation from a context.*
  - **HSA-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
  - **HSA-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
  - **HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*

- **FORMULATING AND SOLVING EQUATIONS FROM WORD PROBLEMS**
  - **HSA-CED.A.3** *Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
  - **HSF-LE.A.2** *Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*
  - **HSF-BF.A.1.a** *Determine an explicit expression, a recursive process, or steps for calculation from a context.*
  - **HSF-LE.A.1.b** *Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.*
  - **HSA-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems.*
  - **HSF-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
  - **HSA-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*

- **FORMULATING AND SOLVING INEQUALITIES FROM WORD PROBLEMS**
  - **HSA-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems.*
  - **HSA-CED.A.3** *Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
  - **HSA-REI.B.3** *Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.*
  - **HSA-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*

2. LITERAL EQUATIONS AND GEOMETRIC SEQUENCES

- **LITERAL EQUATIONS**
  - **HSA-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems.*
  - **HSA-CED.A.4** *Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.*
  - **HSA-REI.B.3** *Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.*

- **SUMS OF GEOMETRIC SEQUENCES**
  - **HSA-SSE.B.4** *Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.*

3. LOGARITHMIC FUNCTIONS AND EXPRESSIONS

- **LOGARITHMIC FUNCTIONS**
  - **HSF-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
  - **HSF-BF.B.4.a** *Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.*
  - **HSF-BF.B.4.c** *Read values of an inverse function from a graph or a table, given that the function has an inverse.*
  - **HSF-BF.B.5** *Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.*
  - **HSF-LE.A.4** *For exponential models, express as a logarithm the solution to ab to the ct power $= d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.*
  - **HSF-IF.A.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.*
  - **HSF-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
  - **HSF-IF.C.7.e** *Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.*

- **EVALUATING LOGARITHMIC EXPRESSIONS**
  - **HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **HSA-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
  - **HSA-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
  - **HSF-LE.A.4** *For exponential models, express as a logarithm the solution to ab to the ct power $= d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.*

4. SOLVING EXPONENTIAL AND LOGARITHMIC EQUATIONS

- **SOLVING EXPONENTIAL EQUATIONS**
  - **HSA-SSE.B.3.c** *Use the properties of exponents to transform expressions for exponential functions.*
  - **HSF-IF.C.8.b** *Use the properties of exponents to interpret expressions for exponential functions.*
  - **HSF-BF.B.4.a** *Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.*
  - **HSF-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
  - **HSF-LE.A.2** *Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*
  - **HSF-IF.C.7.e** *Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.*
  - **HSF-LE.A.4** *For exponential models, express as a logarithm the solution to ab to the ct power $= d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.*

- **SOLVING LOGARITHMIC EQUATIONS**
  - **HSF-IF.A.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each*

element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

- **HSF-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
- **HSF-BF.B.4.a** *Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.*
- **HSF-LE.A.4** *For exponential models, express as a logarithm the solution to ab to the ct power = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.*
- **HSA-REI.A.1** *Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.*

5. POLYNOMIALS 1

- **POLYNOMIAL BASICS**
  - **HSA-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
  - **HSA-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
  - **HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **HSA-APR.A.1** *Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.*

- **ADDITION AND SUBTRACTION OF POLYNOMIALS**
  - **HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **HSA-APR.A.1** *Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.*

6. POLYNOMIALS 2

- **MULTIPLICATION OF POLYNOMIALS**
  - **HSA-APR.A.1** *Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.*
  - **HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*

- **DIVISION OF POLYNOMIALS**
  - **HSA-APR.B.2** *Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).*
  - **HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **HSA-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
  - **HSA-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
  - **HSA-APR.D.6** *Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where (x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.*

7. FACTORING POLYNOMIALS AND THE FACTOR THEOREM

- **FACTORING CUBIC POLYNOMIALS**
  - **HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **HSA-APR.B.3** *Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.*
  - **HSA-APR.C.4** *Prove polynomial identities and use them to describe numerical relationships.*
  - **HSA-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*

- **FACTORING HIGHER-ORDER POLYNOMIALS**

- **HSA-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
- **HSA-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
- **HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
- **HSA-APR.B.3** *Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.*
- **HSA-APR.C.4** *Prove polynomial identities and use them to describe numerical relationships.*

FACTOR THEOREM AND REMAINDER THEOREM

- **HSA-APR.B.2** *Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.*
- **HSA-APR.D.6** *Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where (x), $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.*
- **HSF-IF.A.2** *Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.*

8. POLYNOMIALS AND POLYNOMIAL IDENTITIES

GRAPHS OF POLYNOMIAL FUNCTIONS

- **HSA-APR.B.3** *Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.*
- **HSF-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
- **HSF-BF.B.3** *Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.*

POLYNOMIAL IDENTITIES

- **HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
- **HSA-APR.C.4** *Prove polynomial identities and use them to describe numerical relationships.*
- **HSA-REI.B.4.a** *Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.*
- **HSA-REI.B.4.b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.*
- **HSA-APR.C.5** *Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.*

POLYNOMIAL IDENTITIES AND COMPLEX NUMBERS

- **HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
- **HSA-APR.C.4** *Prove polynomial identities and use them to describe numerical relationships.*
- **HSA-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
- **HSA-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
- **HSN-CN.C.8** *Extend polynomial identities to the complex numbers.*
- **HSA-REI.B.4.b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.*
- **HSN-CN.A.1** *Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.*
- **HSN-CN.C.7** *Solve quadratic equations with real coefficients that have complex solutions.*
- **HSN-CN.C.9** *Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.*

9. QUADRATIC FUNCTIONS AND EQUATIONS

- **REPRESENTATIONS OF QUADRATIC FUNCTIONS**
  - **HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **HSA-REI.B.4.a** *Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.*
  - **HSF-IF.C.8.a** *Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.*
  - **HSF-IF.A.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).*
  - **HSF-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
  - **HSF-IF.C.9** *Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).*
  - **HSA-APR.B.2** *Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by $x - a$ is p(a), so p(a) = 0 if and only if $(x - a)$ is a factor of p(x).*
  - **HSA-APR.B.3** *Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.*
  - **HSA-CED.A.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
  - **HSF-BF.A.1.a** *Determine an explicit expression, a recursive process, or steps for calculation from a context.*
  - **HSA-CED.A.3** *Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
  - **HSA-SSE.B.3.a** *Factor a quadratic expression to reveal the zeros of the function it defines.*
  - **HSF-BF.B.3** *Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.*
  - **HSF-IF.C.7.c** *Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.*

- **COMPLETING THE SQUARE**
  - **HSA-SSE.B.3.b** *Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.*
  - **HSA-REI.B.4.a** *Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.*
  - **HSA-REI.B.4.b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.*
  - **HSF-IF.C.8.a** *Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.*
  - **HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **HSF-IF.C.7.a** *Graph linear and quadratic functions and show intercepts, maxima, and minima.*

10. SQUARE ROOT FUNCTIONS AND EQUATIONS

- **ANALYZING GRAPHS OF SQUARE ROOT FUNCTIONS**
  - **HSF-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
  - **HSF-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
  - **HSF-BF.B.3** *Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.*
  - **HSG-CO.B.6** *Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.*
  - **HSF-IF.C.7.b** *Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.*
  - **HSF-BF.B.4.a** *Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression*

for the inverse.

- **HSF-BF.B.4.c** *Read values of an inverse function from a graph or a table, given that the function has an inverse.*
- **HSF-IF.A.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).*

- **SOLVING SQUARE ROOT EQUATIONS**
  - **HSA-REI.A.2** *Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.*
  - **HSA-REI.A.1** *Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.*
  - **HSF-BF.A.1.a** *Determine an explicit expression, a recursive process, or steps for calculation from a context.*

11. RATIONAL EXPRESSIONS AND EQUATIONS

- **OPERATIONS WITH RATIONAL EXPRESSIONS**
  - **HSA-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
  - **HSA-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
  - **HSA-APR.D.7** *Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.*
  - **HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **HSA-APR.D.6** *Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where (x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.*

- **SOLVING RATIONAL EQUATIONS**
  - **HSA-REI.A.1** *Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.*
  - **HSF-IF.A.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).*
  - **HSA-REI.A.2** *Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.*
  - **HSA-CED.A.3** *Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*

12. RATIONAL FUNCTIONS

- **ANALYZING GRAPHS OF RATIONAL FUNCTIONS**
  - **HSF-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
  - **HSF-BF.B.3** *Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.*
  - **HSF-IF.A.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).*
  - **HSF-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*

- **MODELING SITUATIONS WITH RATIONAL FUNCTIONS**
  - **HSA-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
  - **HSA-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*

- **HSA-CED.A.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
- **HSA-CED.A.3** *Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
- **HSF-BF.A.1.a** *Determine an explicit expression, a recursive process, or steps for calculation from a context.*
- **HSA-REI.A.2** *Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.*

13. TRIGONOMETRY

RADIANS AND THE UNIT CIRCLE

- **HSF-TF.A.1** *Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.*
- **HSF-TF.A.2** *Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.*
- **HSG-C.B.5** *Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.*
- **HSF-TF.A.3** *Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π–x, π+x, and 2π–x in terms of their values for x, where x is any real number.*
- **HSF-TF.A.4** *Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.*
- **HSG-SRT.C.8** *Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*

TRIGONOMETRIC FUNCTIONS

- **HSF-IF.C.7.e** *Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.*
- **HSF-TF.A.2** *Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.*
- **HSF-TF.B.5** *Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*
- **HSF-TF.C.8** *Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.*

LAWS OF SINE AND COSINE

- **HSG-SRT.C.8** *Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*
- **HSG-SRT.D.9** *Derive the formula $A = 1/2\ ab\ \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.*
- **HSG-SRT.D.10** *Prove the Laws of Sines and Cosines and use them to solve problems.*
- **HSG-SRT.D.11** *Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).*

14. FUNCTIONS

DOMAIN AND RANGE

- **HSF-IF.A.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).*
- **HSF-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*

ARITHMETIC OPERATIONS ON FUNCTIONS

- **HSF-BF.A.1.b** *Combine standard function types using arithmetic operations.*

MULTIPLE REPRESENTATIONS OF FUNCTIONS

- **HSA-CED.A.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

- **HSF-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
- **HSF-IF.C.9** *Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).*
- **HSF-LE.A.2** *Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*
- **HSF-LE.A.1.a** *Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.*

15. WORKING WITH FUNCTIONS

INVERSE FUNCTIONS

- **HSF-BF.B.4.c** *Read values of an inverse function from a graph or a table, given that the function has an inverse.*
- **HSF-BF.B.4.a** *Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.*
- **HSF-BF.B.4.d** *Produce an invertible function from a non-invertible function by restricting the domain.*

ABSOLUTE VALUE FUNCTIONS

- **HSF-IF.A.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).*
- **HSF-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
- **HSF-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
- **HSF-BF.B.3** *Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.*
- **HSF-IF.C.7.b** *Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.*

16. PARENT FUNCTIONS AND TRANSFORMATIONS

PARENT FUNCTIONS

- **HSF-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
- **HSF-BF.B.3** *Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.*
- **HSF-IF.C.7.b** *Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.*
- **HSF-IF.C.7.c** *Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.*
- **HSF-IF.A.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).*
- **HSF-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
- **HSF-LE.A.2** *Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*
- **HSF-IF.C.7.e** *Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.*

TRANSFORMATIONS OF PARENT FUNCTIONS

- **HSF-BF.B.3** *Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.*

- **HSF-IF.C.7.b** *Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.*
- **HSF-IF.C.7.c** *Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.*
- **HSF-IF.C.7.e** *Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.*
- **HSG-CO.A.2** *Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).*
- **HSG-CO.B.6** *Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.*
- **HSF-IF.A.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).*
- **HSF-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
- **HSF-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*

- **MULTIPLE TRANSFORMATIONS OF PARENT FUNCTIONS**
  - **HSF-BF.B.3** *Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.*
  - **HSG-CO.A.2** *Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).*
  - **HSG-CO.B.6** *Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.*
  - **HSG-CO.A.5** *Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.*
  - **HSF-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
  - **HSF-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
  - **HSF-IF.C.7.b** *Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.*
  - **HSF-IF.C.7.c** *Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.*
  - **HSF-IF.C.7.e** *Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.*

17. SYSTEMS OF EQUATIONS

- **SOLVING THREE-VARIABLE SYSTEMS OF LINEAR EQUATIONS**
  - **HSA-CED.A.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
  - **HSA-CED.A.3** *Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
  - **HSA-REI.C.5** *Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.*

- **SYSTEMS OF NONLINEAR EQUATIONS**
  - **HSA-REI.C.6** *Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.*
  - **HSA-REI.C.7** *Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.*
  - **HSA-REI.C.5** *Prove that, given a system of two equations in two variables, replacing one equation by the sum of that*

equation and a multiple of the other produces a system with the same solutions.

- **HSA-REI.D.11** *Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*
- **HSA-CED.A.3** *Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
- **HSF-LE.A.2** *Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*

18. SURFACE AREA

SURFACE AREA AND VOLUME OF SPHERES

- **HSG-GMD.A.2** *Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.*
- **HSG-GMD.A.3** *Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*
- **HSG-GMD.B.4** *Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.*
- **HSG-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

SURFACE AREA OF COMPOSITE SOLIDS

- **HSG-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

SURFACE AREA OF SIMILAR SOLIDS

- **HSG-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

19. THREE-DIMENSIONAL GEOMETRY

RELATING TWO-DIMENSIONAL FIGURES TO THREE-DIMENSIONAL SOLIDS

- **HSG-GMD.B.4** *Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.*

MODELING SITUATIONS WITH GEOMETRY

- **HSG-MG.A.3** *Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*
- **HSG-MG.A.2** *Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*

20. STATISTICAL ANALYSIS AND DESIGN

ANALYZING STATISTICAL SAMPLES

- **HSS-IC.A.1** *Understand statistics as a process for making inferences about population parameters based on a random sample from that population.*
- **HSS-IC.B.4** *Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.*
- **HSS-IC.A.2** *Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.*

EXPERIMENTAL AND OBSERVATIONAL DESIGN

- **HSS-IC.B.3** *Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.*

- **CONCLUSIONS IN DATA**
  - **HSS-IC.B.5** *Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.*
  - **HSS-IC.B.6** *Evaluate reports based on data.*

21. STATISTICS AND PROBABILITY

- **NORMAL DISTRIBUTION**
  - **HSS-ID.A.3** *Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).*
  - **HSS-ID.A.4** *Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.*
  - **HSS-IC.B.4** *Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.*

- **ANALYZING DECISIONS IN PROBABILITY**
  - **HSS-MD.B.6** *Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).*
  - **HSS-MD.B.7** *Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).*