Kentucky Tutorials are designed specifically for the Kentucky Academic Standards to prepare students for the K-PREP, EOC exams, ACT, and ACT Plan.

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

1. RATIONAL AND IRRATIONAL NUMBERS

- **OPERATIONS ON RATIONAL AND IRRATIONAL NUMBERS**
  - **HSN-RN.B.3** *Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.*

- **LAWS OF EXPONENTS**
  - **HSN-RN.A.1** *Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.*
  - **HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **HSA-APR.D.6** *Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where (x), $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.*
  - **HSA-REI.A.1** *Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.*
  - **HSN-RN.A.2** *Rewrite expressions involving radicals and rational exponents using the properties of exponents.*

2. EXPRESSIONS AND EQUATIONS 1

- **FORMULATING AND SIMPLIFYING ALGEBRAIC EXPRESSIONS**
  - **HSF-BF.A.1.a** *Determine an explicit expression, a recursive process, or steps for calculation from a context.*
  - **HSA-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
  - **HSA-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
  - **HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*

- **ONE-STEP EQUATIONS AND INEQUALITIES**
  - **HSA-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems.*
  - **HSA-CED.A.3** *Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
  - **HSA-REI.A.1** *Explain each step in solving a simple equation as following from the equality of numbers asserted at the*

previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

- **HSA-REI.B.3** *Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.*
- **HSF-BF.A.1.a** *Determine an explicit expression, a recursive process, or steps for calculation from a context.*

3. EXPRESSIONS AND EQUATIONS 2

MULTI-STEP EQUATIONS AND INEQUALITIES

- **HSA-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems.*
- **HSA-CED.A.3** *Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
- **HSA-REI.A.1** *Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.*
- **HSA-REI.B.3** *Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.*

LITERAL EQUATIONS

- **HSA-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems.*
- **HSA-CED.A.4** *Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.*
- **HSA-REI.B.3** *Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.*

4. FUNCTIONS

FUNCTIONS AND RELATIONS

- **HSF-IF.A.2** *Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.*
- **HSF-IF.A.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).*
- **HSF-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
- **HSF-LE.A.2** *Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*
- **HSF-IF.C.7.b** *Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.*

DOMAIN AND RANGE

- **HSF-IF.A.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).*
- **HSF-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*

MULTIPLE REPRESENTATIONS OF FUNCTIONS

- **HSA-CED.A.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
- **HSF-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
- **HSF-IF.C.9** *Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).*
- **HSF-LE.A.2** *Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*

- **HSF-LE.A.1.a** *Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.*

5. POINTS, LINES, AND ANGLES

- **PARALLEL LINES AND ANGLE RELATIONSHIPS**
  - **HSG-CO.A.1** *Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.*
  - **HSG-CO.C.9** *Prove theorems about lines and angles.*

- **PERPENDICULAR BISECTOR AND ANGLE BISECTOR THEOREMS**
  - **HSG-CO.A.1** *Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.*
  - **HSG-CO.C.10** *Prove theorems about triangles.*
  - **HSG-CO.C.9** *Prove theorems about lines and angles.*

- **CONJECTURES IN COORDINATE GEOMETRY**
  - **HSG-CO.A.2** *Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).*
  - **HSG-CO.C.10** *Prove theorems about triangles.*
  - **HSG-GPE.B.4** *Use coordinates to prove simple geometric theorems algebraically.*

6. CONIC SECTIONS

- **CIRCLES**
  - **HSG-CO.A.1** *Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.*
  - **HSG-GPE.A.1** *Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.*

- **PARABOLAS**
  - **HSG-GMD.B.4** *Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.*
  - **HSG-GPE.A.2** *Derive the equation of a parabola given a focus and directrix.*
  - **HSA-CED.A.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

7. GEOMETRIC TRANSFORMATIONS

- **TRANSFORMATIONS ON THE COORDINATE PLANE**
  - **HSG-CO.A.2** *Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).*
  - **HSG-CO.B.6** *Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.*
  - **HSG-CO.A.5** *Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.*
  - **HSG-CO.A.3** *Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.*
  - **HSG-CO.A.4** *Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.*
  - **HSG-SRT.A.1.b** *The dilation of a line segment is longer or shorter in the ratio given by the scale factor.*

- **HSG-SRT.A.2** *Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.*
- **HSG-SRT.A.1.a** *A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.*

- **DILATIONS, TRANSLATIONS, ROTATIONS, AND REFLECTIONS**
  - **HSG-CO.B.6** *Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.*
  - **HSG-CO.A.5** *Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.*
  - **HSG-CO.A.3** *Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.*
  - **HSG-CO.A.2** *Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).*
  - **HSG-CO.A.4** *Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.*
  - **HSG-SRT.A.2** *Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.*
  - **HSG-SRT.A.1.a** *A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.*
  - **HSG-SRT.A.1.b** *The dilation of a line segment is longer or shorter in the ratio given by the scale factor.*

8. CONGRUENCE AND SIMILARITY

- **TRIANGLES AND CONGRUENCE TRANSFORMATIONS**
  - **HSG-CO.B.6** *Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.*
  - **HSG-CO.B.7** *Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.*
  - **HSG-CO.B.8** *Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.*
  - **HSG-CO.C.10** *Prove theorems about triangles.*
  - **HSG-SRT.B.5** *Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.*

- **TRIANGLES AND SIMILARITY TRANSFORMATIONS**
  - **HSG-CO.C.10** *Prove theorems about triangles.*
  - **HSG-SRT.A.2** *Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.*
  - **HSG-SRT.B.5** *Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.*
  - **HSG-SRT.B.4** *Prove theorems about triangles.*
  - **HSG-SRT.A.3** *Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.*

- **SIMILARITY OF OTHER POLYGONS**
  - **HSG-SRT.A.2** *Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.*

9. TRIANGLES AND QUADRILATERALS

- **TRIANGLE ANGLE THEOREMS**
  - **HSG-CO.C.10** *Prove theorems about triangles.*

- **TRIANGLE BISECTORS**
  - **HSG-CO.C.10** *Prove theorems about triangles.*
  - **HSG-CO.C.9** *Prove theorems about lines and angles.*
  - **HSG-SRT.B.5** *Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.*
  - **HSG-SRT.B.4** *Prove theorems about triangles.*
  - **HSG-CO.D.12** *Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).*
  - **HSG-C.A.3** *Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.*

- **MEDIANS AND ALTITUDES OF TRIANGLES**
  - **HSG-CO.C.10** *Prove theorems about triangles.*

10. QUADRILATERALS AND CONSTRUCTIONS

- **PARALLELOGRAMS AND RECTANGLES**
  - **HSG-CO.C.11** *Prove theorems about parallelograms.*
  - **HSG-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

- **SQUARES AND RHOMBI**
  - **HSG-CO.C.11** *Prove theorems about parallelograms.*
  - **HSG-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

- **CONSTRUCTIONS**
  - **HSG-CO.D.12** *Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).*
  - **HSG-CO.D.13** *Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.*
  - **HSG-C.A.4** *Construct a tangent line from a point outside a given circle to the circle.*

11. RIGHT TRIANGLES AND TRIGONOMETRIC RATIOS

- **THE PYTHAGOREAN THEOREM**
  - **HSG-SRT.C.8** *Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*
  - **HSG-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*
  - **HSG-SRT.B.5** *Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.*
  - **HSG-SRT.B.4** *Prove theorems about triangles.*
  - **HSG-CO.C.10** *Prove theorems about triangles.*

- **TRIGONOMETRIC RATIOS**
  - **HSF-TF.A.3** *Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π–x, π+x, and 2π–x in terms of their values for x, where x is any real number.*

- **HSG-SRT.C.8** *Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*
- **HSG-SRT.C.6** *Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.*
- **HSG-SRT.C.7** *Explain and use the relationship between the sine and cosine of complementary angles.*
- **HSG-SRT.B.5** *Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.*
- **HSG-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

12. TRIGONOMETRY

RADIANS AND THE UNIT CIRCLE

- **HSF-TF.A.1** *Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.*
- **HSF-TF.A.2** *Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.*
- **HSG-C.B.5** *Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.*
- **HSF-TF.A.3** *Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π–x, π+x, and 2π–x in terms of their values for x, where x is any real number.*
- **HSF-TF.A.4** *Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.*
- **HSG-SRT.C.8** *Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*

TRIGONOMETRIC FUNCTIONS

- **HSF-IF.C.7.e** *Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.*
- **HSF-TF.A.2** *Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.*
- **HSF-TF.B.5** *Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*
- **HSF-TF.C.8** *Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.*

13. CIRCLES 1

CIRCLE BASICS

- **HSG-CO.A.1** *Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.*
- **HSG-C.A.2** *Identify and describe relationships among inscribed angles, radii, and chords.*

CENTRAL ANGLES, INSCRIBED ANGLES, AND CHORDS

- **HSG-C.A.2** *Identify and describe relationships among inscribed angles, radii, and chords.*
- **HSG-CO.A.1** *Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.*
- **HSG-CO.C.9** *Prove theorems about lines and angles.*
- **HSG-C.A.3** *Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.*
- **HSG-C.B.5** *Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.*

14. CIRCLES 2

SECANTS, ANGLES, AND INTERCEPTED ARCS

- **HSG-CO.A.1** *Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.*

- **HSG-CO.C.9** *Prove theorems about lines and angles.*
- **HSG-C.A.2** *Identify and describe relationships among inscribed angles, radii, and chords.*

TANGENTS, ANGLES, AND INTERCEPTED ARCS

- **HSG-CO.A.1** *Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.*
- **HSG-CO.C.9** *Prove theorems about lines and angles.*
- **HSG-C.A.2** *Identify and describe relationships among inscribed angles, radii, and chords.*

15. ADVANCED CIRCLE PROPERTIES

CONGRUENT AND SIMILAR CIRCLES

- **HSG-CO.B.6** *Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.*
- **HSG-CO.A.1** *Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.*
- **HSG-C.A.1** *Prove that all circles are similar.*
- **HSG-CO.A.4** *Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.*
- **HSG-SRT.A.2** *Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.*

CIRCUMFERENCE AND ARC LENGTH

- **HSG-CO.A.1** *Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.*
- **HSG-GMD.A.1** *Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.*
- **HSG-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

AREA OF CIRCLES AND SECTORS

- **HSG-GMD.A.1** *Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.*
- **HSG-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*
- **HSG-CO.A.1** *Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.*
- **HSG-C.B.5** *Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.*

16. EXPONENTIAL FUNCTIONS, EQUATIONS, AND INEQUALITIES

EXPONENTIAL FUNCTIONS

- **HSA-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
- **HSA-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
- **HSF-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
- **HSF-IF.C.8.b** *Use the properties of exponents to interpret expressions for exponential functions.*
- **HSF-LE.A.1.a** *Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.*
- **HSF-IF.B.6** *Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
- **HSF-LE.A.2** *Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a*

description of a relationship, or two input-output pairs (include reading these from a table).

- **HSF-LE.A.3** *Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.*
- **HSF-IF.A.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).*
- **HSA-SSE.B.3.c** *Use the properties of exponents to transform expressions for exponential functions.*
- **HSF-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
- **HSF-IF.C.7.e** *Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.*
- **HSA-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems.*
- **HSA-CED.A.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
- **HSA-REI.B.3** *Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.*
- **HSF-LE.B.5** *Interpret the parameters in a linear or exponential function in terms of a context.*
- **HSF-BF.A.1.a** *Determine an explicit expression, a recursive process, or steps for calculation from a context.*
- **HSF-LE.A.1.c** *Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.*

EXPONENTIAL GROWTH AND DECAY

- **HSA-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
- **HSA-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
- **HSF-LE.A.2** *Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*
- **HSF-LE.B.5** *Interpret the parameters in a linear or exponential function in terms of a context.*
- **HSF-IF.C.8.b** *Use the properties of exponents to interpret expressions for exponential functions.*
- **HSF-LE.A.1.a** *Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.*
- **HSF-LE.A.1.c** *Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.*
- **HSF-LE.A.3** *Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.*
- **HSF-LE.A.1.b** *Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.*
- **HSA-CED.A.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

SOLVING EXPONENTIAL INEQUALITIES

- **HSF-LE.A.2** *Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*
- **HSA-CED.A.3** *Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
- **HSA-CED.A.1** *Create equations and inequalities in one variable and use them to solve problems.*
- **HSA-CED.A.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
- **HSA-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
- **HSF-LE.A.1.c** *Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.*

17. POLYNOMIALS 1

POLYNOMIAL BASICS

- **HSA-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
- **HSA-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*

- **HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
- **HSA-APR.A.1** *Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.*

- **ADDITION AND SUBTRACTION OF POLYNOMIALS**
  - **HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **HSA-APR.A.1** *Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.*

18. POLYNOMIALS 2

- **MULTIPLICATION OF POLYNOMIALS**
  - **HSA-APR.A.1** *Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.*
  - **HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*

- **ARITHMETIC OPERATIONS ON FUNCTIONS**
  - **HSF-BF.A.1.b** *Combine standard function types using arithmetic operations.*

19. FACTORING

- **FACTORING QUADRATIC TRINOMIALS**
  - **HSA-SSE.B.3.a** *Factor a quadratic expression to reveal the zeros of the function it defines.*
  - **HSA-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
  - **HSA-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
  - **HSA-REI.B.4.b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.*

- **FACTORING SPECIAL CASES**
  - **HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **HSA-APR.C.4** *Prove polynomial identities and use them to describe numerical relationships.*
  - **HSA-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
  - **HSA-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
  - **HSA-APR.B.3** *Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.*

20. COMPLEX NUMBERS

- **COMPLEX NUMBERS**
  - **HSN-CN.A.1** *Know there is a complex number i such that $i^2 = –1$, and every complex number has the form $a + bi$ with a and b real.*
  - **HSN-CN.A.2** *Use the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.*

- **POLYNOMIAL IDENTITIES AND COMPLEX NUMBERS**
  - **HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **HSA-APR.C.4** *Prove polynomial identities and use them to describe numerical relationships.*
  - **HSA-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
  - **HSA-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
  - **HSN-CN.C.8** *Extend polynomial identities to the complex numbers.*
  - **HSA-REI.B.4.b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the*

quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

- **HSN-CN.A.1** *Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.*
- **HSN-CN.C.7** *Solve quadratic equations with real coefficients that have complex solutions.*
- **HSN-CN.C.9** *Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.*

21. REPRESENTATIONS OF QUADRATIC FUNCTIONS

QUADRATIC FUNCTIONS

- **HSF-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
- **HSF-IF.C.7.a** *Graph linear and quadratic functions and show intercepts, maxima, and minima.*
- **HSA-CED.A.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
- **HSA-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
- **HSA-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
- **HSA-SSE.B.3.b** *Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.*
- **HSF-IF.C.8.a** *Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.*
- **HSF-BF.A.1.a** *Determine an explicit expression, a recursive process, or steps for calculation from a context.*

ANALYZING GRAPHS OF QUADRATIC FUNCTIONS

- **HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
- **HSF-IF.C.9** *Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).*
- **HSF-IF.A.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.*
- **HSF-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
- **HSF-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
- **HSF-IF.C.7.a** *Graph linear and quadratic functions and show intercepts, maxima, and minima.*
- **HSF-BF.B.3** *Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.*
- **HSA-APR.B.3** *Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.*
- **HSA-REI.B.4.b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.*
- **HSF-IF.C.8.a** *Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.*

REPRESENTATIONS OF QUADRATIC FUNCTIONS

- **HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
- **HSA-REI.B.4.a** *Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.*
- **HSF-IF.C.8.a** *Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.*
- **HSF-IF.A.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.*
- **HSF-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in*

terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

- **HSF-IF.C.9** *Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).*
- **HSA-APR.B.2** *Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.*
- **HSA-APR.B.3** *Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.*
- **HSA-CED.A.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
- **HSF-BF.A.1.a** *Determine an explicit expression, a recursive process, or steps for calculation from a context.*
- **HSA-CED.A.3** *Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
- **HSA-SSE.B.3.a** *Factor a quadratic expression to reveal the zeros of the function it defines.*
- **HSF-BF.B.3** *Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.*
- **HSF-IF.C.7.c** *Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.*

22. QUADRATIC PARENT FUNCTIONS AND TRANSFORMATIONS

QUADRATIC PARENT FUNCTION

- **HSF-IF.C.7.a** *Graph linear and quadratic functions and show intercepts, maxima, and minima.*
- **HSA-REI.D.10** *Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).*
- **HSF-IF.A.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.*
- **HSF-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*

TRANSFORMATIONS OF THE QUADRATIC PARENT FUNCTION

- **HSF-BF.B.3** *Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.*
- **HSG-CO.A.2** *Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).*
- **HSF-IF.A.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.*
- **HSF-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*

23. SOLVING QUADRATIC EQUATIONS

SOLVING QUADRATIC EQUATIONS BY FACTORING

- **HSA-SSE.B.3.a** *Factor a quadratic expression to reveal the zeros of the function it defines.*
- **HSA-REI.B.4.b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.*
- **HSF-IF.C.8.a** *Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.*
- **HSA-APR.B.3** *Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.*
- **HSA-APR.C.4** *Prove polynomial identities and use them to describe numerical relationships.*
- **HSF-IF.A.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$*

denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

- **HSF-IF.C.7.a** *Graph linear and quadratic functions and show intercepts, maxima, and minima.*
- **HSA-CED.A.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
- **HSF-BF.A.1.a** *Determine an explicit expression, a recursive process, or steps for calculation from a context.*

- **COMPLETING THE SQUARE**
  - **HSA-SSE.B.3.b** *Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.*
  - **HSA-REI.B.4.a** *Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.*
  - **HSA-REI.B.4.b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.*
  - **HSF-IF.C.8.a** *Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.*
  - **HSA-SSE.A.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **HSF-IF.C.7.a** *Graph linear and quadratic functions and show intercepts, maxima, and minima.*

24. QUADRATIC FORMULA AND COMPLEX NUMBERS

- **QUADRATIC FORMULA**
  - **HSA-SSE.A.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
  - **HSA-SSE.A.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
  - **HSA-REI.B.4.b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.*
  - **HSF-IF.A.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).*
  - **HSF-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
  - **HSA-CED.A.3** *Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
  - **HSA-REI.B.4.a** *Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.*
  - **HSF-BF.A.1.a** *Determine an explicit expression, a recursive process, or steps for calculation from a context.*

- **COMPLEX NUMBERS AND QUADRATIC FUNCTIONS**
  - **HSA-REI.B.4.b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.*
  - **HSN-CN.C.7** *Solve quadratic equations with real coefficients that have complex solutions.*
  - **HSN-CN.A.1** *Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.*
  - **HSN-CN.A.2** *Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.*

25. NONLINEAR FUNCTIONS

- **INVERSE FUNCTIONS**
  - **HSF-BF.B.4.c** *Read values of an inverse function from a graph or a table, given that the function has an inverse.*
  - **HSF-BF.B.4.a** *Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.*
  - **HSF-BF.B.4.d** *Produce an invertible function from a non-invertible function by restricting the domain.*

- **ABSOLUTE VALUE FUNCTIONS**
  - **HSF-IF.A.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).*
  - **HSF-IF.B.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
  - **HSF-IF.B.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
  - **HSF-BF.B.3** *Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.*
  - **HSF-IF.C.7.b** *Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.*

- **SYSTEMS OF NONLINEAR EQUATIONS**
  - **HSA-REI.C.6** *Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.*
  - **HSA-REI.C.7** *Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.*
  - **HSA-REI.C.5** *Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.*
  - **HSA-REI.D.11** *Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*
  - **HSA-CED.A.3** *Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
  - **HSF-LE.A.2** *Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*

26. VOLUME

- **VOLUME OF PRISMS AND PYRAMIDS**
  - **HSG-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*
  - **HSG-GMD.A.2** *Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.*
  - **HSG-GMD.A.1** *Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.*
  - **HSG-GMD.A.3** *Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*
  - **HSG-GMD.B.4** *Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.*

- **VOLUME OF CYLINDERS AND CONES**
  - **HSG-GMD.A.1** *Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.*
  - **HSG-GMD.A.2** *Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.*
  - **HSG-GMD.A.3** *Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*
  - **HSG-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*
  - **HSG-GMD.B.4** *Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.*

- **VOLUME OF COMPOSITE SOLIDS**

- **HSG-MG.A.1** *Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*
- **HSG-GMD.A.3** *Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*
- **HSG-GMD.A.2** *Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.*

27. BASIC PROBABILITY CONCEPTS

- **INTRODUCTION TO PROBABILITY**
  - **HSS-CP.A.2** *Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.*
  - **HSS-CP.B.8** *Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.*
  - **HSS-CP.A.5** *Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.*
  - **HSS-CP.A.1** *Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").*
  - **HSS-CP.B.7** *Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.*

- **COMBINATIONS AND PERMUTATIONS**
  - **HSS-CP.B.9** *Use permutations and combinations to compute probabilities of compound events and solve problems.*

28. ADVANCED PROBABILITY CONCEPTS

- **CONDITIONAL PROBABILITY**
  - **HSS-CP.A.3** *Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.*
  - **HSS-CP.A.5** *Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.*
  - **HSS-CP.B.6** *Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.*
  - **HSS-CP.A.2** *Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.*
  - **HSS-ID.B.5** *Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.*
  - **HSS-CP.A.4** *Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.*

- **GEOMETRIC PROBABILITIES**
  - **HSG-MG.A.3** *Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*
  - **HSS-MD.B.7** *Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).*
  - **HSS-CP.A.1** *Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").*
  - **HSS-CP.B.7** *Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.*

- **ANALYZING DECISIONS IN PROBABILITY**
  - **HSS-MD.B.6** *Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).*
  - **HSS-MD.B.7** *Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a*

hockey goalie at the end of a game).