California Tutorials are designed specifically for the California Common Core State Standards and the California Next Generation Science Standards to prepare students for the Smarter Balanced Assessment Consortium (SBAC) exams and the California Science Tests.

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

1. REAL NUMBER SYSTEM

- **LAWS OF EXPONENTS**
  - **N-RN.1** *Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.*
  - **A-SSE.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **N-RN.2** *Rewrite expressions involving radicals and rational exponents using the properties of exponents.*

- **OPERATIONS ON RATIONAL AND IRRATIONAL NUMBERS**
  - **N-RN.3** *Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.*

- **MONITORING PRECISION AND ACCURACY**
  - **N-Q.1** *Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.*
  - **N-Q.2** *Define appropriate quantities for the purpose of descriptive modeling.*
  - **N-Q.3** *Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.*

2. EQUATIONS AND INEQUALITIES

- **ONE-STEP EQUATIONS AND INEQUALITIES**
  - **A-CED.1** *Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.*
  - **A-CED.3** *Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
  - **A-REI.3** *Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.*
  - **F-BF.1.a** *Determine an explicit expression, a recursive process, or steps for calculation from a context.*

- **MULTI-STEP EQUATIONS AND INEQUALITIES**

- **A-CED.3** *Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
- **A-CED.1** *Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.*
- **A-REI.3** *Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.*
- **A-REI.1** *Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.*

- **AXIOMS OF EQUALITY**
  - **A-SSE.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **A-REI.1** *Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.*

- **LITERAL EQUATIONS**
  - **A-REI.3** *Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.*
  - **A-CED.1** *Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.*
  - **A-CED.4** *Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.*

3. WRITING EXPRESSIONS AND EQUATIONS

- **FORMULATING AND SIMPLIFYING ALGEBRAIC EXPRESSIONS**
  - **F-BF.1.a** *Determine an explicit expression, a recursive process, or steps for calculation from a context.*
  - **A-SSE.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*
  - **A-SSE.2** *Use the structure of an expression to identify ways to rewrite it.*

- **FORMULATING AND SOLVING EQUATIONS FROM WORD PROBLEMS**
  - **F-LE.2** *Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*
  - **F-LE.1.b** *Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.*
  - **A-CED.1** *Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.*

- **FORMULATING AND SOLVING INEQUALITIES FROM WORD PROBLEMS**
  - **A-CED.3** *Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
  - **A-CED.1** *Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.*
  - **A-REI.3** *Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.*

4. FUNCTIONS

- **FUNCTIONS AND RELATIONS**
  - **F-IF.2** *Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.*
  - **F-IF.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).*
  - **F-IF.7.b** *Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.*

- **DOMAIN AND RANGE**

- **DOMAIN AND RANGE**
  - **F-IF.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*

- **EVALUATING FUNCTIONS**
  - **F-IF.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).*
  - **F-IF.2** *Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.*

5. GRAPHS OF LINEAR EQUATIONS AND INEQUALITIES

- **SLOPE**
  - **F-IF.6** *Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
  - **F-IF.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).*
  - **F-IF.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*

- **GRAPHING AND ANALYZING LINEAR FUNCTIONS**
  - **F-IF.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).*
  - **F-LE.2** *Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*
  - **F-IF.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
  - **F-IF.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
  - **F-IF.6** *Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

- **GRAPHING AND MANIPULATING Y = MX + B**
  - **F-IF.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).*
  - **F-LE.2** *Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*
  - **F-IF.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
  - **F-IF.6** *Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
  - **F-LE.1.b** *Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.*
  - **F-LE.5** *Interpret the parameters in a linear or exponential function in terms of a context.*
  - **F-IF.7.a** *Graph linear and quadratic functions and show intercepts, maxima, and minima.*

- **GRAPHS OF LINEAR INEQUALITIES**
  - **A-REI.12** *Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.*
  - **A-CED.3** *Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*

6. LINEAR EQUATIONS

- **SLOPE-INTERCEPT FORM OF A LINEAR EQUATION**
  - **A-REI.10** *Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).*
  - **F-IF.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).*
  - **F-IF.7.a** *Graph linear and quadratic functions and show intercepts, maxima, and minima.*
  - **F-LE.2** *Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*
  - **F-IF.6** *Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

- **POINT-SLOPE FORM OF A LINEAR EQUATION**
  - **A-REI.10** *Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).*
  - **F-IF.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).*
  - **F-LE.2** *Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*
  - **F-IF.7.a** *Graph linear and quadratic functions and show intercepts, maxima, and minima.*

7. LINEAR SYSTEMS

- **SOLVING SYSTEMS OF LINEAR EQUATIONS: GUESS AND CHECK**
  - **A-CED.3** *Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
  - **F-BF.1.a** *Determine an explicit expression, a recursive process, or steps for calculation from a context.*
  - **A-REI.6** *Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.*

- **SOLVING SYSTEMS OF LINEAR EQUATIONS: GRAPHING**
  - **A-CED.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
  - **A-CED.3** *Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
  - **A-REI.6** *Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.*
  - **A-REI.11** *Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

- **SOLVING SYSTEMS OF LINEAR EQUATIONS: SUBSTITUTION**
  - **A-CED.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
  - **A-CED.3** *Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
  - **A-REI.6** *Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.*

- **SOLVING SYSTEMS OF LINEAR EQUATIONS: ELIMINATION**
  - **A-CED.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on*

coordinate axes with labels and scales.

- **A-CED.3** *Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
- **A-REI.5** *Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.*
- **A-REI.6** *Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.*

- **SOLVING SYSTEMS OF LINEAR INEQUALITIES**
  - **A-CED.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
  - **A-CED.3** *Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
  - **A-REI.12** *Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.*

- **SOLVING THREE-VARIABLE SYSTEMS OF LINEAR EQUATIONS**
  - **A-CED.3** *Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
  - **A-REI.5** *Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.*

8. EXPONENTIAL FUNCTIONS, EQUATIONS, AND INEQUALITIES

- **EXPONENTIAL FUNCTIONS**
  - **A-SSE.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
  - **A-SSE.3.c** *Use the properties of exponents to transform expressions for exponential functions.*
  - **F-IF.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
  - **F-IF.8.b** *Use the properties of exponents to interpret expressions for exponential functions.*
  - **F-LE.1.a** *Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.*
  - **F-IF.6** *Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
  - **F-IF.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).*
  - **F-IF.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
  - **F-IF.7.e** *Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.*
  - **F-LE.1.c** *Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.*
  - **F-LE.2** *Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*
  - **F-LE.5** *Interpret the parameters in a linear or exponential function in terms of a context.*
  - **A-CED.1** *Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.*

- **EXPONENTIAL GROWTH AND DECAY**
  - **F-IF.8.b** *Use the properties of exponents to interpret expressions for exponential functions.*
  - **F-LE.1.a** *Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.*
  - **F-LE.1.c** *Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.*

- **F-LE.5** *Interpret the parameters in a linear or exponential function in terms of a context.*
- **A-SSE.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
- **F-LE.2** *Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*
- **F-LE.3** *Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.*
- **F-LE.1.b** *Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.*
- **A-CED.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

SOLVING EXPONENTIAL INEQUALITIES

- **A-CED.3** *Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
- **A-SSE.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*
- **A-CED.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
- **F-LE.1.c** *Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.*

9. SEQUENCES

SEQUENCES

- **F-BF.1.a** *Determine an explicit expression, a recursive process, or steps for calculation from a context.*
- **F-BF.2** *Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*
- **F-IF.3** *Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.*
- **F-LE.2** *Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*

ARITHMETIC AND GEOMETRIC SEQUENCES

- **F-BF.2** *Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*
- **F-IF.3** *Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.*
- **F-LE.2** *Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*
- **F-BF.1.a** *Determine an explicit expression, a recursive process, or steps for calculation from a context.*

10. POLYNOMIALS

POLYNOMIAL BASICS

- **A-SSE.1.a** *Interpret parts of an expression, such as terms, factors, and coefficients.*

ADDITION AND SUBTRACTION OF POLYNOMIALS

- **A-APR.1** *Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.*

MULTIPLICATION OF POLYNOMIALS

- **A-APR.1** *Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.*

11. FACTORING

FACTORING QUADRATIC TRINOMIALS

- **A-SSE.3.a** *Factor a quadratic expression to reveal the zeros of the function it defines.*
- **A-REI.4.b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.*

- **FACTORING SPECIAL CASES**
  - **A-SSE.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **A-SSE.1.b** *Interpret complicated expressions by viewing one or more of their parts as a single entity.*

- **FACTORING HIGHER-ORDER POLYNOMIALS**
  - **A-SSE.2** *Use the structure of an expression to identify ways to rewrite it.*

12. GRAPHS OF QUADRATIC FUNCTIONS

- **QUADRATIC FUNCTIONS**
  - **F-IF.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
  - **F-LE.6** *Apply quadratic functions to physical problems, such as the motion of an object under the force of gravity.*

- **ANALYZING GRAPHS OF QUADRATIC FUNCTIONS**
  - **F-IF.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
  - **F-IF.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.*
  - **F-IF.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
  - **F-IF.7.a** *Graph linear and quadratic functions and show intercepts, maxima, and minima.*
  - **A-REI.4.b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.*
  - **F-IF.9** *Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).*

- **REPRESENTATIONS OF QUADRATIC FUNCTIONS**
  - **A-SSE.2** *Use the structure of an expression to identify ways to rewrite it.*
  - **A-REI.4.a** *Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.*
  - **F-IF.8.a** *Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.*
  - **A-CED.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
  - **F-IF.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.*
  - **A-SSE.3.a** *Factor a quadratic expression to reveal the zeros of the function it defines.*
  - **F-IF.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
  - **F-LE.6** *Apply quadratic functions to physical problems, such as the motion of an object under the force of gravity.*

13. SOLVING QUADRATIC FUNCTIONS

- **SOLVING QUADRATIC FUNCTIONS WITH FACTORING**
  - **A-SSE.3.a** *Factor a quadratic expression to reveal the zeros of the function it defines.*

- **A-REI.4.b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.*
- **F-IF.8.a** *Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.*
- **F-LE.6** *Apply quadratic functions to physical problems, such as the motion of an object under the force of gravity.*
- **F-IF.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).*
- **F-IF.7.a** *Graph linear and quadratic functions and show intercepts, maxima, and minima.*

COMPLETING THE SQUARE

- **A-REI.4.a** *Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.*
- **A-REI.4.b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.*
- **F-IF.8.a** *Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.*
- **A-SSE.3.b** *Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.*
- **F-IF.7.a** *Graph linear and quadratic functions and show intercepts, maxima, and minima.*

QUADRATIC FORMULA

- **F-IF.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
- **A-REI.4.b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.*
- **F-IF.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).*
- **A-CED.3** *Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*
- **A-REI.4.a** *Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.*
- **F-BF.1.a** *Determine an explicit expression, a recursive process, or steps for calculation from a context.*

14. PARENT FUNCTIONS AND TRANSFORMATIONS

LINEAR AND EXPONENTIAL PARENT FUNCTIONS

- **A-REI.10** *Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).*
- **F-IF.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
- **F-IF.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
- **F-IF.7.e** *Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.*
- **F-LE.1.c** *Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.*

QUADRATIC PARENT FUNCTION

- **A-REI.10** *Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).*

- **TRANSFORMATIONS OF THE LINEAR AND EXPONENTIAL PARENT FUNCTIONS**
  - **F-BF.3** *Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.*

- **TRANSFORMATIONS OF THE QUADRATIC PARENT FUNCTION**
  - **F-BF.3** *Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.*
  - **F-IF.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*

15. NONLINEAR FUNCTIONS

- **LINEAR VERSUS NONLINEAR FUNCTIONS**
  - **F-LE.1.a** *Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.*
  - **F-LE.1.b** *Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.*
  - **F-IF.9** *Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).*
  - **F-LE.2** *Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*
  - **F-IF.6** *Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
  - **F-IF.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
  - **F-LE.1.c** *Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.*

- **ABSOLUTE VALUE FUNCTIONS**
  - **F-IF.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
  - **F-IF.1** *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).*
  - **F-IF.4** *For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*
  - **F-IF.7.b** *Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.*
  - **F-BF.3** *Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.*

- **SYSTEMS OF NONLINEAR EQUATIONS**
  - **A-REI.6** *Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.*
  - **A-REI.7** *Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.*
  - **A-REI.11** *Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*
  - **A-REI.5** *Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.*
  - **A-CED.3** *Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.*

16. WORKING WITH FUNCTIONS

- **ARITHMETIC OPERATIONS ON FUNCTIONS**
  - **F-BF.1.b** *Combine standard function types using arithmetic operations.*

- **MULTIPLE REPRESENTATIONS OF FUNCTIONS**
  - **A-CED.2** *Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
  - **F-IF.9** *Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).*
  - **F-LE.1.a** *Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.*

- **INVERSE FUNCTIONS**
  - **F-BF.4.a** *Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.*

17. STATISTICS

- **DATA ANALYSIS**
  - **S-ID.1** *Represent data with plots on the real number line (dot plots, histograms, and box plots).*
  - **S-ID.2** *Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.*
  - **S-ID.3** *Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).*

- **FREQUENCY TABLES**
  - **S-ID.5** *Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.*
  - **S-ID.3** *Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).*

- **SCATTERPLOTS**
  - **S-ID.6.a** *Fit a function to the data; use functions fitted to data to solve problems in the context of the data.*
  - **S-ID.6.b** *Informally assess the fit of a function by plotting and analyzing residuals.*
  - **S-ID.6.c** *Fit a linear function for a scatter plot that suggests a linear association.*
  - **S-ID.9** *Distinguish between correlation and causation.*
  - **S-ID.7** *Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.*

- **SCATTERPLOTS AND MODELING**
  - **S-ID.6.a** *Fit a function to the data; use functions fitted to data to solve problems in the context of the data.*
  - **S-ID.6.b** *Informally assess the fit of a function by plotting and analyzing residuals.*
  - **S-ID.6.c** *Fit a linear function for a scatter plot that suggests a linear association.*
  - **S-ID.8** *Compute (using technology) and interpret the correlation coefficient of a linear fit.*
  - **S-ID.7** *Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.*
  - **F-LE.1.a** *Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.*
  - **F-LE.1.c** *Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.*