

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

This Tutorial is aligned with the Common Core State Standards for Mathematics.

## 1. POINTS AND LINES

### • POINTS, RAYS, LINE SEGMENTS, LINES, AND FIGURES

- **CCSS.Math.Content.HSG-CO.A.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
- **CCSS.Math.Content.HSG-MG.A.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

### • PARALLEL AND PERPENDICULAR LINES

- **CCSS.Math.Content.HSG-GPE.B.5** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
- **CCSS.Math.Content.HSG-CO.A.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

## 2. LINES AND ANGLES

### • PARALLEL LINES AND ANGLE RELATIONSHIPS

- **CCSS.Math.Content.HSG-CO.A.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
- **CCSS.Math.Content.HSG-CO.C.9** Prove theorems about lines and angles.

### • PERPENDICULAR BISECTOR AND ANGLE BISECTOR THEOREMS

- **CCSS.Math.Content.HSG-CO.A.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
- **CCSS.Math.Content.HSG-CO.C.9** Prove theorems about lines and angles.
- **CCSS.Math.Content.HSG-CO.C.10** Prove theorems about triangles.

## 3. INTRODUCTION TO COORDINATE GEOMETRY

- **SLOPE-INTERCEPT FORM OF A LINEAR EQUATION**

- **CCSS.Math.Content.HSS-ID.C.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- **CCSS.Math.Content.HSA-REI.D.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- **CCSS.Math.Content.HSF-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **CCSS.Math.Content.HSF-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- **CCSS.Math.Content.HSF-IF.C.7a** Graph linear and quadratic functions and show intercepts, maxima, and minima.
- **CCSS.Math.Content.HSG-GPE.B.5** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
- **CCSS.Math.Content.HSF-IF.B.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

- **LENGTH AND THE DISTANCE FORMULA**

- **CCSS.Math.Content.HSG-GPE.B.6** Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
- **CCSS.Math.Content.HSG-GPE.B.7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.
- **CCSS.Math.Content.HSG-MG.A.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

## 4. COORDINATE GEOMETRY

- **MIDPOINT FORMULA ON THE COORDINATE PLANE**

- **CCSS.Math.Content.HSG-GPE.B.6** Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
- **CCSS.Math.Content.HSG-MG.A.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
- **CCSS.Math.Content.HSG-GPE.B.7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

- **CONJECTURES IN COORDINATE GEOMETRY**

- **CCSS.Math.Content.HSG-CO.C.10** Prove theorems about triangles.
- **CCSS.Math.Content.HSG-GPE.B.4** Use coordinates to prove simple geometric theorems algebraically.

## 5. PERIMETER AND AREA ON THE PLANE

- **PERIMETER ON THE COORDINATE PLANE**

- **CCSS.Math.Content.HSG-GPE.B.7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.
- **CCSS.Math.Content.HSG-MG.A.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
- **CCSS.Math.Content.HSG-GPE.B.4** Use coordinates to prove simple geometric theorems algebraically.

- **AREA ON THE COORDINATE PLANE**

- **CCSS.Math.Content.HSG-GPE.B.7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.
- **CCSS.Math.Content.HSG-MG.A.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

## 6. TRANSFORMATIONS ON THE COORDINATE PLANE

### • TRANSFORMATIONS ON THE COORDINATE PLANE

- **CCSS.Math.Content.HSG-CO.A.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
- **CCSS.Math.Content.HSG-CO.B.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- **CCSS.Math.Content.HSG-CO.A.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
- **CCSS.Math.Content.HSG-CO.A.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
- **CCSS.Math.Content.HSG-SRT.A.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
- **CCSS.Math.Content.HSG-SRT.A.1 a** A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- **CCSS.Math.Content.HSG-SRT.A.1 b** The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
- **CCSS.Math.Content.HSG-CO.A.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

### • DILATIONS, TRANSLATIONS, ROTATIONS, AND REFLECTIONS

- **CCSS.Math.Content.HSG-CO.B.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- **CCSS.Math.Content.HSG-CO.A.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
- **CCSS.Math.Content.HSG-CO.A.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
- **CCSS.Math.Content.HSG-CO.A.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
- **CCSS.Math.Content.HSG-CO.A.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
- **CCSS.Math.Content.HSG-SRT.A.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
- **CCSS.Math.Content.HSG-SRT.A.1 a** A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- **CCSS.Math.Content.HSG-SRT.A.1 b** The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

## 7. CONGRUENCE

### • TRIANGLES AND CONGRUENCE TRANSFORMATIONS

- **CCSS.Math.Content.HSG-CO.B.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- **CCSS.Math.Content.HSG-CO.B.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
- **CCSS.Math.Content.HSG-CO.B.8** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
- **CCSS.Math.Content.HSG-CO.C.10** Prove theorems about triangles.
- **CCSS.Math.Content.HSG-SRT.B.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

- **CONGRUENCE OF OTHER POLYGONS**

- **CCSS.Math.Content.HSG-MG.A.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
- **CCSS.Math.Content.HSG-CO.B.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- **CCSS.Math.Content.HSG-CO.A.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
- **CCSS.Math.Content.HSG-CO.A.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
- **CCSS.Math.Content.HSG-CO.A.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

## 8. SIMILARITY

- **TRIANGLES AND SIMILARITY TRANSFORMATIONS**

- **CCSS.Math.Content.HSG-CO.C.10** Prove theorems about triangles.
- **CCSS.Math.Content.HSG-SRT.A.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
- **CCSS.Math.Content.HSG-SRT.A.3** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
- **CCSS.Math.Content.HSG-SRT.B.4** Prove theorems about triangles.
- **CCSS.Math.Content.HSG-SRT.B.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

- **SIMILARITY OF OTHER POLYGONS**

- **CCSS.Math.Content.HSG-SRT.A.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

## 9. TRIANGLES

- **TRIANGLE ANGLE THEOREMS**

- **CCSS.Math.Content.HSG-CO.C.10** Prove theorems about triangles.

- **TRIANGLE BISECTORS**

- **CCSS.Math.Content.HSG-CO.C.9** Prove theorems about lines and angles.

- **CCSS.Math.Content.HSG-CO.C.10** Prove theorems about triangles.
- **CCSS.Math.Content.HSG-SRT.B.4** Prove theorems about triangles.
- **CCSS.Math.Content.HSG-SRT.B.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
- **CCSS.Math.Content.HSG-CO.D.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).
- **CCSS.Math.Content.HSG-C.A.3** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

- **MEDIANS AND ALTITUDES OF TRIANGLES**

- **CCSS.Math.Content.HSG-CO.C.10** Prove theorems about triangles.

## 10. QUADRILATERALS AND CONSTRUCTIONS

- **PARALLELOGRAMS AND RECTANGLES**

- **CCSS.Math.Content.HSG-CO.C.11** Prove theorems about parallelograms.
- **CCSS.Math.Content.HSG-MG.A.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

- **SQUARES AND RHOMBI**

- **CCSS.Math.Content.HSG-CO.C.11** Prove theorems about parallelograms.
- **CCSS.Math.Content.HSG-MG.A.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

- **CONSTRUCTIONS**

- **CCSS.Math.Content.HSG-CO.D.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).
- **CCSS.Math.Content.HSG-CO.D.13** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
- **CCSS.Math.Content.HSG-C.A.4** Construct a tangent line from a point outside a given circle to the circle.

## 11. RIGHT TRIANGLES

- **PYTHAGOREAN THEOREM**

- **CCSS.Math.Content.HSG-SRT.C.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
- **CCSS.Math.Content.HSG-MG.A.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
- **CCSS.Math.Content.HSG-SRT.B.4** Prove theorems about triangles.
- **CCSS.Math.Content.HSG-SRT.B.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
- **CCSS.Math.Content.HSG-CO.C.10** Prove theorems about triangles.

- **TRIGONOMETRIC RATIOS**

- **CCSS.Math.Content.HSF-TF.A.3** Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\pi/3$ ,  $\pi/4$  and  $\pi/6$ , and use the unit circle to express the values of sine, cosine, and tangent for  $\pi-x$ ,  $\pi+x$ , and  $2\pi-x$  in terms of their values for  $x$ , where  $x$  is any real number.
- **CCSS.Math.Content.HSG-SRT.C.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
- **CCSS.Math.Content.HSG-SRT.C.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

- **CCSS.Math.Content.HSG-SRT.C.7** Explain and use the relationship between the sine and cosine of complementary angles.
- **CCSS.Math.Content.HSG-SRT.B.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
- **CCSS.Math.Content.HSG-MG.A.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

## 12. TRIANGLES AND TRIGONOMETRY

### • LAWS OF SINE AND COSINE

- **CCSS.Math.Content.HSG-SRT.C.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
- **CCSS.Math.Content.HSG-SRT.D.9** Derive the formula  $A = 1/2 ab \sin(C)$  for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
- **CCSS.Math.Content.HSG-SRT.D.10** Prove the Laws of Sines and Cosines and use them to solve problems.
- **CCSS.Math.Content.HSG-SRT.D.11** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

### • RADIANS AND THE UNIT CIRCLE

- **CCSS.Math.Content.HSF-TF.A.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- **CCSS.Math.Content.HSF-TF.A.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- **CCSS.Math.Content.HSG-C.B.5** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
- **CCSS.Math.Content.HSF-TF.A.3** Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\pi/3$ ,  $\pi/4$  and  $\pi/6$ , and use the unit circle to express the values of sine, cosine, and tangent for  $\pi-x$ ,  $\pi+x$ , and  $2\pi-x$  in terms of their values for  $x$ , where  $x$  is any real number.
- **CCSS.Math.Content.HSF-TF.A.4** Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
- **CCSS.Math.Content.HSG-SRT.C.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

## 13. CIRCLES AND ANGLES 1

### • CIRCLE BASICS

- **CCSS.Math.Content.HSG-CO.A.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
- **CCSS.Math.Content.HSG-C.A.2** Identify and describe relationships among inscribed angles, radii, and chords.

### • CENTRAL ANGLES, INSCRIBED ANGLES, AND CHORDS

- **CCSS.Math.Content.HSG-C.A.2** Identify and describe relationships among inscribed angles, radii, and chords.
- **CCSS.Math.Content.HSG-CO.A.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
- **CCSS.Math.Content.HSG-CO.C.9** Prove theorems about lines and angles.
- **CCSS.Math.Content.HSG-C.A.3** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
- **CCSS.Math.Content.HSG-C.B.5** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.



## 14. CIRCLES AND ANGLES 2

### • SECANTS, ANGLES, AND INTERCEPTED ARCS

- **CCSS.Math.Content.HSG-CO.A.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
- **CCSS.Math.Content.HSG-CO.C.9** Prove theorems about lines and angles.
- **CCSS.Math.Content.HSG-C.A.2** Identify and describe relationships among inscribed angles, radii, and chords.

### • TANGENTS, ANGLES, AND INTERCEPTED ARCS

- **CCSS.Math.Content.HSG-CO.A.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
- **CCSS.Math.Content.HSG-CO.C.9** Prove theorems about lines and angles.
- **CCSS.Math.Content.HSG-C.A.2** Identify and describe relationships among inscribed angles, radii, and chords.

## 15. PROPERTIES OF CIRCLES

### • CONGRUENT AND SIMILAR CIRCLES

- **CCSS.Math.Content.HSG-CO.B.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- **CCSS.Math.Content.HSG-CO.A.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
- **CCSS.Math.Content.HSG-C.A.1** Prove that all circles are similar.
- **CCSS.Math.Content.HSG-CO.A.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
- **CCSS.Math.Content.HSG-SRT.A.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

### • CIRCUMFERENCE AND ARC LENGTH

- **CCSS.Math.Content.HSG-CO.A.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
- **CCSS.Math.Content.HSG-GMD.A.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
- **CCSS.Math.Content.HSG-MG.A.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

### • AREA OF CIRCLES AND SECTORS

- **CCSS.Math.Content.HSG-GMD.A.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
- **CCSS.Math.Content.HSG-MG.A.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
- **CCSS.Math.Content.HSG-CO.A.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
- **CCSS.Math.Content.HSG-C.B.5** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

## 16. CONIC SECTIONS

### • CIRCLES

- **CCSS.Math.Content.HSG-CO.A.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
- **CCSS.Math.Content.HSG-GPE.A.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

- **PARABOLAS**

- **CCSS.Math.Content.HSG-GMD.B.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
- **CCSS.Math.Content.HSG-GPE.A.2** Derive the equation of a parabola given a focus and directrix.
- **CCSS.Math.Content.HSA-CED.A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

## 17. SURFACE AREA

- **SURFACE AREA AND VOLUME OF SPHERES**

- **CCSS.Math.Content.HSG-GMD.A.2** Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
- **CCSS.Math.Content.HSG-GMD.A.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
- **CCSS.Math.Content.HSG-GMD.B.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
- **CCSS.Math.Content.HSG-MG.A.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

- **SURFACE AREA OF COMPOSITE SOLIDS**

- **CCSS.Math.Content.HSG-MG.A.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

- **SURFACE AREA OF SIMILAR SOLIDS**

- **CCSS.Math.Content.HSG-MG.A.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

## 18. VOLUME 1

- **RELATING TWO-DIMENSIONAL FIGURES TO THREE-DIMENSIONAL SOLIDS**

- **CCSS.Math.Content.HSG-GMD.B.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

- **VOLUME OF PRISMS AND PYRAMIDS**

- **CCSS.Math.Content.HSG-GMD.A.2** Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
- **CCSS.Math.Content.HSG-MG.A.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
- **CCSS.Math.Content.HSG-GMD.A.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
- **CCSS.Math.Content.HSG-GMD.A.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
- **CCSS.Math.Content.HSG-GMD.B.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

## 19. VOLUME 2



## • VOLUME OF CYLINDERS AND CONES

- **CCSS.Math.Content.HSG-GMD.A.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
- **CCSS.Math.Content.HSG-GMD.A.2** Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
- **CCSS.Math.Content.HSG-GMD.A.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
- **CCSS.Math.Content.HSG-MG.A.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
- **CCSS.Math.Content.HSG-GMD.B.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

## • MODELING SITUATIONS WITH GEOMETRY

- **CCSS.Math.Content.HSG-MG.A.2** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
- **CCSS.Math.Content.HSG-MG.A.3** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

## 20. VOLUME OF SIMILAR AND COMPOSITE SHAPES

### • VOLUME OF COMPOSITE SOLIDS

- **CCSS.Math.Content.HSG-MG.A.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
- **CCSS.Math.Content.HSG-GMD.A.2** Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
- **CCSS.Math.Content.HSG-GMD.A.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

### • VOLUME OF SIMILAR SOLIDS

- **CCSS.Math.Content.HSG-GMD.A.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
- **CCSS.Math.Content.HSG-MG.A.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
- **CCSS.Math.Content.HSG-GMD.A.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

## 21. BASIC PROBABILITY CONCEPTS

### • INTRODUCTION TO PROBABILITY

- **CCSS.Math.Content.HSS-CP.A.2** Understand that two events  $A$  and  $B$  are independent if the probability of  $A$  and  $B$  occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
- **CCSS.Math.Content.HSS-CP.B.8** Apply the general Multiplication Rule in a uniform probability model,  $P(A \text{ and } B) = (A)P(B|A) = P(B)P(A|B)$ , and interpret the answer in terms of the model.
- **CCSS.Math.Content.HSS-CP.A.5** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
- **CCSS.Math.Content.HSS-CP.A.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
- **CCSS.Math.Content.HSS-CP.B.7** Apply the Addition Rule,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , and interpret the answer in terms of the model.

### • COMBINATIONS AND PERMUTATIONS

- **CCSS.Math.Content.HSS-CP.B.9** Use permutations and combinations to compute probabilities of compound events and solve problems.

## 22. ADVANCED PROBABILITY CONCEPTS

### • CONDITIONAL PROBABILITY

- **CCSS.Math.Content.HSS-CP.A.3** Understand the conditional probability of  $A$  given  $B$  as  $P(A \text{ and } B)/P(B)$ , and interpret independence of  $A$  and  $B$  as saying that the conditional probability of  $A$  given  $B$  is the same as the probability of  $A$ , and the conditional probability of  $B$  given  $A$  is the same as the probability of  $B$ .
- **CCSS.Math.Content.HSS-CP.A.5** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
- **CCSS.Math.Content.HSS-CP.B.6** Find the conditional probability of  $A$  given  $B$  as the fraction of  $B$ 's outcomes that also belong to  $A$ , and interpret the answer in terms of the model.
- **CCSS.Math.Content.HSS-CP.A.2** Understand that two events  $A$  and  $B$  are independent if the probability of  $A$  and  $B$  occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
- **CCSS.Math.Content.HSS-ID.B.5** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
- **CCSS.Math.Content.HSS-CP.A.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

### • GEOMETRIC PROBABILITIES

- **CCSS.Math.Content.HSG-MG.A.3** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
- **CCSS.Math.Content.HSS-MD.B.7** Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).
- **CCSS.Math.Content.HSS-CP.A.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
- **CCSS.Math.Content.HSS-CP.B.7** Apply the Addition Rule,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , and interpret the answer in terms of the model.

### • ANALYZING DECISIONS IN PROBABILITY

- **CCSS.Math.Content.HSS-MD.B.6** Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
- **CCSS.Math.Content.HSS-MD.B.7** Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).