

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

This Tutorial is aligned with the Common Core State Standards for Mathematics.

## 1. THE NUMBER SYSTEM

### ● RATIONAL AND IRRATIONAL NUMBERS

- **CCSS.Math.Content.8.NS.A.1** Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

### ● APPROXIMATING IRRATIONAL NUMBERS

- **CCSS.Math.Content.8.NS.A.2** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g.,  $\pi^2$ ).
- **CCSS.Math.Content.8.EE.A.2** Use square root and cube root symbols to represent solutions to equations of the form  $x^2 = p$  and  $x^3 = p$ , where  $p$  is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that  $\sqrt{2}$  is irrational.
- **CCSS.Math.Content.8.NS.A.1** Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

## 2. EXPONENTS

### ● PROPERTIES OF EXPONENTS

- **CCSS.Math.Content.8.EE.A.1** Know and apply the properties of integer exponents to generate equivalent numerical expressions.

### ● POWERS OF 10

- **CCSS.Math.Content.8.EE.A.3** Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.
- **CCSS.Math.Content.8.EE.A.4** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

### ● SCIENTIFIC NOTATION

- **CCSS.Math.Content.8.EE.A.3** Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.
- **CCSS.Math.Content.8.EE.A.4** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

### 3. PROPORTIONAL REASONING AND SLOPE

#### ● SLOPE

- **CCSS.Math.Content.8.EE.B.6** Use similar triangles to explain why the slope  $m$  is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation  $y = mx$  for a line through the origin and the equation  $y = mx + b$  for a line intercepting the vertical axis at  $b$ .
- **CCSS.Math.Content.8.F.B.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two  $(x, y)$  values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- **CCSS.Math.Content.8.F.B.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
- **CCSS.Math.Content.8.EE.B.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

#### ● MULTIPLE REPRESENTATIONS OF PROPORTIONS

- **CCSS.Math.Content.8.EE.B.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.
- **CCSS.Math.Content.8.F.B.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two  $(x, y)$  values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- **CCSS.Math.Content.8.F.B.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

### 4. FUNCTIONS

#### ● RELATIONS AND FUNCTIONS

- **CCSS.Math.Content.8.F.A.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

#### ● COMPARING FUNCTIONS

- **CCSS.Math.Content.8.F.A.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- **CCSS.Math.Content.8.F.B.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two  $(x, y)$  values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

#### ● GRAPHS OF FUNCTIONS

- **CCSS.Math.Content.8.F.A.3** Interpret the equation  $y = mx + b$  as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.
- **CCSS.Math.Content.8.F.B.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two  $(x, y)$  values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- **CCSS.Math.Content.8.F.B.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph

(e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

## 5. LINEAR FUNCTIONS

### • SLOPE-INTERCEPT FORM

- **CCSS.Math.Content.8.EE.B.6** Use similar triangles to explain why the slope  $m$  is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation  $y = mx$  for a line through the origin and the equation  $y = mx + b$  for a line intercepting the vertical axis at  $b$ .
- **CCSS.Math.Content.8.F.A.3** Interpret the equation  $y = mx + b$  as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.
- **CCSS.Math.Content.8.F.B.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two  $(x, y)$  values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

### • WRITING LINEAR FUNCTIONS

- **CCSS.Math.Content.8.F.B.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two  $(x, y)$  values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

## 6. SOLVING EQUATIONS

### • SOLVING LINEAR EQUATIONS

- **CCSS.Math.Content.8.EE.C.7a** Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form  $x = a$ ,  $a = a$ , or  $a = b$  results (where  $a$  and  $b$  are different numbers).
- **CCSS.Math.Content.8.EE.C.7b** Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
- **CCSS.Math.Content.8.EE.C.8b** Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.
- **CCSS.Math.Content.8.EE.C.8c** Solve real-world and mathematical problems leading to two linear equations in two variables.

### • SOLVING SYSTEMS OF LINEAR EQUATIONS

- **CCSS.Math.Content.8.EE.C.8a** Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
- **CCSS.Math.Content.8.EE.C.8b** Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.
- **CCSS.Math.Content.8.EE.C.8c** Solve real-world and mathematical problems leading to two linear equations in two variables.

### • SOLVING EQUATIONS USING ROOTS

- **CCSS.Math.Content.8.EE.A.2** Use square root and cube root symbols to represent solutions to equations of the form  $x^2 = p$  and  $x^3 = p$ , where  $p$  is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that  $\sqrt{2}$  is irrational.

## 7. THE PYTHAGOREAN THEOREM AND DISTANCE FORMULA

### • THE PYTHAGOREAN THEOREM

- **CCSS.Math.Content.8.G.B.6** Explain a proof of the Pythagorean Theorem and its converse.
- **CCSS.Math.Content.8.G.B.7** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

- **THE CONVERSE OF THE PYTHAGOREAN THEOREM**

- **CCSS.Math.Content.8.G.B.6** Explain a proof of the Pythagorean Theorem and its converse.
- **CCSS.Math.Content.8.G.B.7** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

- **DISTANCE ON THE COORDINATE PLANE**

- **CCSS.Math.Content.8.G.B.8** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

## 8. THREE-DIMENSIONAL GEOMETRY

- **VOLUME OF CYLINDERS AND CONES**

- **CCSS.Math.Content.8.G.C.9** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

- **SPHERES**

- **CCSS.Math.Content.8.G.C.9** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

## 9. TRANSFORMATIONS, CONGRUENCE, AND SIMILARITY 1

- **BASICS OF TRANSFORMATIONS**

- **CCSS.Math.Content.8.G.A.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
- **CCSS.Math.Content.8.G.A.4** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
- **CCSS.Math.Content.8.G.A.1b** Angles are taken to angles of the same measure.
- **CCSS.Math.Content.8.G.A.1a** Lines are taken to lines, and line segments to line segments of the same length.
- **CCSS.Math.Content.8.G.A.1c** Parallel lines are taken to parallel lines.

- **TRANSFORMATIONS AND CONGRUENCE**

- **CCSS.Math.Content.8.G.A.2** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
- **CCSS.Math.Content.8.G.A.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

## 10. TRANSFORMATIONS, CONGRUENCE, AND SIMILARITY 2

- **TRANSFORMATIONS ON THE COORDINATE PLANE**

- **CCSS.Math.Content.8.G.A.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
- **CCSS.Math.Content.8.G.A.4** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

- **SIMILARITY AND DILATIONS**

- **CCSS.Math.Content.8.G.A.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
- **CCSS.Math.Content.8.G.A.4** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures,

describe a sequence that exhibits the similarity between them.

- **CCSS.Math.Content.8.G.B.7** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

## 11. ANGLES AND ANGLE RELATIONSHIPS

### ● PARALLEL LINES AND ANGLE RELATIONSHIPS

- **CCSS.Math.Content.8.G.A.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

### ● ANGLE RELATIONSHIPS IN TRIANGLES

- **CCSS.Math.Content.8.G.A.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

## 12. DATA AND STATISTICS

### ● SCATTERPLOTS

- **CCSS.Math.Content.8.SP.A.1** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
- **CCSS.Math.Content.8.SP.A.2** Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

### ● LINEAR MODELS IN DATA

- **CCSS.Math.Content.8.SP.A.1** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
- **CCSS.Math.Content.8.SP.A.2** Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
- **CCSS.Math.Content.8.SP.A.3** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.
- **CCSS.Math.Content.8.F.B.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two  $(x, y)$  values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

### ● FREQUENCY TABLES

- **CCSS.Math.Content.8.SP.A.4** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.