Georgia Tutorials are designed specifically for the Georgia Standards of Excellence and the Georgia Performance Standards to prepare students for the Georgia Milestones.

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible to students through focused content, modeled logic and process, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students focus on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

1. EXPRESSIONS AND EQUATIONS

- **LAWS OF EXPONENTS**
  - **MGSE9-12.A.SSE.2** *Use the structure of an expression to rewrite it in different equivalent forms.*
  - **MGSE9-12.N.RN.2** *Rewrite expressions involving radicals and rational exponents using the properties of exponents.*
  - **MGSE9-12.N.RN.1** *Explain how the meaning of rational exponents follows from extending the properties of integer exponents to rational numbers, allowing for a notation for radicals in terms of rational exponents.*
  - **MGSE9-12.A.APR.6** *Rewrite simple rational expressions in different forms using inspection, long division, or a computer algebra system; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$.*

- **LITERAL EQUATIONS**
  - **MGSE9-12.A.CED.1** *Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, simple rational, and exponential functions.*
  - **MGSE9-12.A.CED.4** *Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations.*

2. APPLYING EQUATIONS AND INEQUALITIES

- **FORMULATING AND SOLVING EQUATIONS FROM WORD PROBLEMS**
  - **MGSE9-12.A.CED.1** *Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, simple rational, and exponential functions.*
  - **MGSE9-12.F.IF.4** *Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*
  - **MGSE9-12.A.SSE.1a** *Interpret parts of an expression, such as terms, factors, and coefficients, in context.*

- **FORMULATING AND SOLVING INEQUALITIES FROM WORD PROBLEMS**
  - **MGSE9-12.A.CED.3** *Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.*
  - **MGSE9-12.A.CED.1** *Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, simple rational, and exponential functions.*

- **SUMS OF GEOMETRIC SEQUENCES**
  - *MGSE9-12.A.SSE.4* *Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.*

3. FUNCTIONS AND THEIR INVERSE

- **FUNCTIONS AND RELATIONS**
  - *MGSE9-12.F.IF.4* *Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*
  - *MGSE9-12.F.IF.7b* *Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.*

- **DOMAIN AND RANGE**
  - *MGSE9-12.F.IF.5* *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*

- **INVERSE FUNCTIONS**
  - *MGSE9-12.F.BF.4c* *Read values of an inverse function from a graph or a table, given that the function has an inverse.*
  - *MGSE9-12.F.BF.4a* *Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.*

4. LINEAR FUNCTIONS, EQUATIONS, AND INEQUALITIES

- **SLOPE**
  - *MGSE9-12.F.IF.6* *Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
  - *MGSE9-12.F.IF.4* *Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*

- **GRAPHING AND ANALYZING LINEAR FUNCTIONS**
  - *MGSE9-12.F.IF.5* *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
  - *MGSE9-12.F.IF.4* *Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*

- **GRAPHING AND MANIPULATING Y = MX + B**
  - *MGSE9-12.F.IF.6* *Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
  - *MGSE9-12.A.CED.2* *Create linear, quadratic, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
  - *MGSE9-12.F.IF.4* *Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*

5. EXPONENTIAL FUNCTIONS

- **EXPONENTIAL FUNCTIONS**
  - *MGSE9-12.F.IF.8b* *Use the properties of exponents to interpret expressions for exponential functions.*
  - *MGSE9-12.A.SSE.1a* *Interpret parts of an expression, such as terms, factors, and coefficients, in context.*
  - *MGSE9-12.A.SSE.1b* *Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the*

meaning (in context) of individual terms or factors.

- **MGSE9-12.F.IF.6** *Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
- **MGSE9-12.F.IF.7e** *Graph exponential and logarithmic functions, showing intercepts and end behavior.*
- **MGSE9-12.F.IF.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
- **MGSE9-12.F.IF.4** *Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*
- **MGSE9-12.A.CED.2** *Create linear, quadratic, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
- **MGSE9-12.A.CED.1** *Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, simple rational, and exponential functions.*
- **MGSE9-12.A.SSE.3c** *Use the properties of exponents to transform expressions for exponential functions.*

- **EXPONENTIAL GROWTH AND DECAY**
  - **MGSE9-12.F.IF.8b** *Use the properties of exponents to interpret expressions for exponential functions.*
  - **MGSE9-12.A.SSE.1a** *Interpret parts of an expression, such as terms, factors, and coefficients, in context.*
  - **MGSE9-12.A.SSE.1b** *Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.*
  - **MGSE9-12.A.CED.2** *Create linear, quadratic, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

6. LOGARITHMIC FUNCTIONS

- **LOGARITHMIC FUNCTIONS**
  - **MGSE9-12.F.BF.4c** *Read values of an inverse function from a graph or a table, given that the function has an inverse.*
  - **MGSE9-12.F.BF.5** *Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.*
  - **MGSE9-12.F.BF.4a** *Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.*
  - **MGSE9-12.F.IF.4** *Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*
  - **MGSE9-12.F.IF.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
  - **MGSE9-12.F.IF.7e** *Graph exponential and logarithmic functions, showing intercepts and end behavior.*

- **EVALUATING LOGARITHMIC EXPRESSIONS**
  - **MGSE9-12.A.SSE.1a** *Interpret parts of an expression, such as terms, factors, and coefficients, in context.*
  - **MGSE9-12.A.SSE.1b** *Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.*
  - **MGSE9-12.A.SSE.2** *Use the structure of an expression to rewrite it in different equivalent forms.*
  - **MGSE9-12.F.LE.4** *For exponential models, express as a logarithm the solution to ab to the ct power = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.*

7. SOLVING EXPONENTIAL AND LOGARITHMIC EQUATIONS AND INEQUALITIES

- **SOLVING EXPONENTIAL EQUATIONS**
  - **MGSE9-12.F.BF.5** *Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.*
  - **MGSE9-12.F.BF.4a** *Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.*
  - **MGSE9-12.F.IF.8b** *Use the properties of exponents to interpret expressions for exponential functions.*
  - **MGSE9-12.F.LE.4** *For exponential models, express as a logarithm the solution to ab to the ct power = d where a, c,*

and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology.

- ○ **MGSE9-12.A.SSE.3c** *Use the properties of exponents to transform expressions for exponential functions.*
- ○ **MGSE9-12.F.IF.7e** *Graph exponential and logarithmic functions, showing intercepts and end behavior.*
- ○ **MGSE9-12.F.IF.4** *Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*
- ○ **MGSE9-12.A.REI.11** *Using graphs, tables, or successive approximations, show that the solution to the equation f(x) = g(x) is the x-value where the y-values of f(x) and g(x) are the same.*

- **SOLVING LOGARITHMIC EQUATIONS**
  - ○ **MGSE9-12.F.BF.5** *Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.*
  - ○ **MGSE9-12.F.BF.4a** *Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.*
  - ○ **MGSE9-12.F.LE.4** *For exponential models, express as a logarithm the solution to ab to the ct power = d where a, c, and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology.*
  - ○ **MGSE9-12.A.REI.11** *Using graphs, tables, or successive approximations, show that the solution to the equation f(x) = g(x) is the x-value where the y-values of f(x) and g(x) are the same.*
  - ○ **MGSE9-12.F.IF.4** *Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*

- **SOLVING EXPONENTIAL INEQUALITIES**
  - ○ **MGSE9-12.A.CED.2** *Create linear, quadratic, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
  - ○ **MGSE9-12.A.CED.3** *Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.*
  - ○ **MGSE9-12.A.CED.1** *Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, simple rational, and exponential functions.*

8. POLYNOMIALS

- **POLYNOMIAL BASICS**
  - ○ **MGSE9-12.A.SSE.1a** *Interpret parts of an expression, such as terms, factors, and coefficients, in context.*
  - ○ **MGSE9-12.A.SSE.1b** *Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.*
  - ○ **MGSE9-12.A.APR.1** *Add, subtract, and multiply polynomials; understand that polynomials form a system analogous to the integers in that they are closed under these operations.*
  - ○ **MGSE9-12.A.SSE.2** *Use the structure of an expression to rewrite it in different equivalent forms.*

- **ADDITION AND SUBTRACTION OF POLYNOMIALS**
  - ○ **MGSE9-12.A.APR.1** *Add, subtract, and multiply polynomials; understand that polynomials form a system analogous to the integers in that they are closed under these operations.*
  - ○ **MGSE9-12.A.SSE.2** *Use the structure of an expression to rewrite it in different equivalent forms.*

9. MULTIPLICATION AND DIVISION OF POLYNOMIALS

- **MULTIPLICATION OF POLYNOMIALS**
  - ○ **MGSE9-12.A.APR.1** *Add, subtract, and multiply polynomials; understand that polynomials form a system analogous to the integers in that they are closed under these operations.*
  - ○ **MGSE9-12.A.SSE.2** *Use the structure of an expression to rewrite it in different equivalent forms.*

- **DIVISION OF POLYNOMIALS**

- **MGSE9-12.A.APR.6** *Rewrite simple rational expressions in different forms using inspection, long division, or a computer algebra system; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x).*
- **MGSE9-12.A.SSE.1a** *Interpret parts of an expression, such as terms, factors, and coefficients, in context.*
- **MGSE9-12.A.SSE.1b** *Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.*
- **MGSE9-12.A.SSE.2** *Use the structure of an expression to rewrite it in different equivalent forms.*

10. SOLVING QUADRATIC EQUATIONS

- ### SOLVING QUADRATIC EQUATIONS BY FACTORING
  - **MGSE9-12.A.REI.4b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, factoring, completing the square, and the quadratic formula, as appropriate to the initial form of the equation.*
  - **MGSE9-12.A.APR.4** *Prove polynomial identities and use them to describe numerical relationships.*
  - **MGSE9-12.A.APR.3** *Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.*
  - **MGSE9-12.A.REI.11** *Using graphs, tables, or successive approximations, show that the solution to the equation f(x) = g(x) is the x-value where the y-values of f(x) and g(x) are the same.*
  - **MGSE9-12.A.CED.2** *Create linear, quadratic, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

- ### COMPLETING THE SQUARE
  - **MGSE9-12.A.REI.4b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, factoring, completing the square, and the quadratic formula, as appropriate to the initial form of the equation.*
  - **MGSE9-12.A.SSE.2** *Use the structure of an expression to rewrite it in different equivalent forms.*

- ### QUADRATIC FORMULA
  - **MGSE9-12.A.SSE.1a** *Interpret parts of an expression, such as terms, factors, and coefficients, in context.*
  - **MGSE9-12.A.SSE.1b** *Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.*
  - **MGSE9-12.A.REI.4b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, factoring, completing the square, and the quadratic formula, as appropriate to the initial form of the equation.*
  - **MGSE9-12.N.CN.7** *Solve quadratic equations with real coefficients that have complex solutions by (but not limited to) square roots, completing the square, and the quadratic formula.*
  - **MGSE9-12.F.IF.4** *Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*

11. QUADRATIC FUNCTIONS

- ### ANALYZING GRAPHS OF QUADRATIC FUNCTIONS
  - **MGSE9-12.F.IF.9** *Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).*
  - **MGSE9-12.A.SSE.2** *Use the structure of an expression to rewrite it in different equivalent forms.*
  - **MGSE9-12.F.IF.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
  - **MGSE9-12.F.IF.4** *Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*
  - **MGSE9-12.A.REI.11** *Using graphs, tables, or successive approximations, show that the solution to the equation f(x) = g(x) is the x-value where the y-values of f(x) and g(x) are the same.*
  - **MGSE9-12.A.APR.3** *Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.*

- ### COMPLEX NUMBERS AND QUADRATIC FUNCTIONS

- MGSE9-12.A.REI.4b *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, factoring, completing the square, and the quadratic formula, as appropriate to the initial form of the equation.*
- MGSE9-12.N.CN.7 *Solve quadratic equations with real coefficients that have complex solutions by (but not limited to) square roots, completing the square, and the quadratic formula.*
- MGSE9-12.N.CN.1 *Understand there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b are real numbers.*
- MGSE9-12.N.CN.2 *Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.*

- **REPRESENTATIONS OF QUADRATIC FUNCTIONS**
  - MGSE9-12.A.SSE.2 *Use the structure of an expression to rewrite it in different equivalent forms.*
  - MGSE9-12.F.IF.9 *Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).*
  - MGSE9-12.F.IF.4 *Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*
  - MGSE9-12.A.CED.2 *Create linear, quadratic, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
  - MGSE9-12.A.CED.3 *Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.*

12. FACTORING POLYNOMIALS I

- **GRAPHS OF POLYNOMIAL FUNCTIONS**
  - MGSE9-12.F.IF.7c *Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.*
  - MGSE9-12.F.IF.4 *Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*
  - MGSE9-12.A.APR.3 *Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.*

- **FACTORING SPECIAL CASES**
  - MGSE9-12.A.APR.4 *Prove polynomial identities and use them to describe numerical relationships.*
  - MGSE9-12.A.SSE.1a *Interpret parts of an expression, such as terms, factors, and coefficients, in context.*
  - MGSE9-12.A.SSE.1b *Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.*
  - MGSE9-12.A.SSE.2 *Use the structure of an expression to rewrite it in different equivalent forms.*
  - MGSE9-12.A.APR.3 *Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.*

- **FACTORING CUBIC POLYNOMIALS**
  - MGSE9-12.A.APR.4 *Prove polynomial identities and use them to describe numerical relationships.*
  - MGSE9-12.A.SSE.2 *Use the structure of an expression to rewrite it in different equivalent forms.*
  - MGSE9-12.A.APR.3 *Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.*
  - MGSE9-12.A.SSE.1b *Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.*

13. FACTORING POLYNOMIALS II

- **FACTORING HIGHER-ORDER POLYNOMIALS**
  - MGSE9-12.A.APR.3 *Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.*

- ○ **MGSE9-12.A.SSE.1a** *Interpret parts of an expression, such as terms, factors, and coefficients, in context.*
- ○ **MGSE9-12.A.SSE.1b** *Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.*
- ○ **MGSE9-12.N.CN.9** *Use the Fundamental Theorem of Algebra to find all roots of a polynomial equation.*
- ○ **MGSE9-12.A.APR.4** *Prove polynomial identities and use them to describe numerical relationships.*
- ○ **MGSE9-12.A.SSE.2** *Use the structure of an expression to rewrite it in different equivalent forms.*

- ● **FACTOR THEOREM AND REMAINDER THEOREM**
  - ○ **MGSE9-12.A.APR.6** *Rewrite simple rational expressions in different forms using inspection, long division, or a computer algebra system; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$.*
  - ○ **MGSE9-12.A.APR.2** *Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.*

14. COMPLEX NUMBERS AND POLYNOMIAL IDENTITIES

- ● **COMPLEX NUMBERS**
  - ○ **MGSE9-12.N.CN.1** *Understand there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b are real numbers.*
  - ○ **MGSE9-12.N.CN.2** *Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.*

- ● **POLYNOMIAL IDENTITIES**
  - ○ **MGSE9-12.A.APR.4** *Prove polynomial identities and use them to describe numerical relationships.*
  - ○ **MGSE9-12.A.SSE.2** *Use the structure of an expression to rewrite it in different equivalent forms.*
  - ○ **MGSE9-12.A.REI.4b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, factoring, completing the square, and the quadratic formula, as appropriate to the initial form of the equation.*
  - ○ **MGSE9-12.A.APR.5** *Know and apply that the Binomial Theorem gives the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.*

- ● **POLYNOMIAL IDENTITIES AND COMPLEX NUMBERS**
  - ○ **MGSE9-12.A.APR.4** *Prove polynomial identities and use them to describe numerical relationships.*
  - ○ **MGSE9-12.A.SSE.1a** *Interpret parts of an expression, such as terms, factors, and coefficients, in context.*
  - ○ **MGSE9-12.A.SSE.1b** *Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.*
  - ○ **MGSE9-12.A.SSE.2** *Use the structure of an expression to rewrite it in different equivalent forms.*
  - ○ **MGSE9-12.N.CN.8** *Extend polynomial identities to include factoring with complex numbers.*
  - ○ **MGSE9-12.A.REI.4b** *Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, factoring, completing the square, and the quadratic formula, as appropriate to the initial form of the equation.*
  - ○ **MGSE9-12.N.CN.1** *Understand there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b are real numbers.*
  - ○ **MGSE9-12.N.CN.7** *Solve quadratic equations with real coefficients that have complex solutions by (but not limited to) square roots, completing the square, and the quadratic formula.*
  - ○ **MGSE9-12.N.CN.9** *Use the Fundamental Theorem of Algebra to find all roots of a polynomial equation.*
  - ○ **MGSE9-12.N.CN.3** *Find the conjugate of a complex number; use the conjugate to find the quotient of complex numbers.*

15. SQUARE ROOT EQUATIONS AND FUNCTIONS

- ● **ANALYZING GRAPHS OF SQUARE ROOT FUNCTIONS**
  - ○ **MGSE9-12.F.BF.3** *Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
  - ○ **MGSE9-12.F.IF.7b** *Graph square root, cube root, and piecewise-defined functions, including step functions and absolute*

- MGSE9-12.F.IF.4 *Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*
- MGSE9-12.F.BF.4c *Read values of an inverse function from a graph or a table, given that the function has an inverse.*
- MGSE9-12.F.BF.4a *Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.*
- MGSE9-12.F.IF.5 *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*

- ### SOLVING SQUARE ROOT EQUATIONS

    - MGSE9-12.A.REI.2 *Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.*
    - MGSE9-12.A.REI.11 *Using graphs, tables, or successive approximations, show that the solution to the equation f(x) = g(x) is the x-value where the y-values of f(x) and g(x) are the same.*

16. RATIONAL EXPRESSIONS AND FUNCTIONS

- ### OPERATIONS WITH RATIONAL EXPRESSIONS

    - MGSE9-12.A.APR.7 *Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.*
    - MGSE9-12.A.SSE.1a *Interpret parts of an expression, such as terms, factors, and coefficients, in context.*
    - MGSE9-12.A.SSE.1b *Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.*
    - MGSE9-12.A.APR.6 *Rewrite simple rational expressions in different forms using inspection, long division, or a computer algebra system; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x).*
    - MGSE9-12.A.SSE.2 *Use the structure of an expression to rewrite it in different equivalent forms.*

- ### ANALYZING GRAPHS OF RATIONAL FUNCTIONS

    - MGSE9-12.F.BF.3 *Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
    - MGSE9-12.F.IF.7d *Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.*
    - MGSE9-12.F.IF.4 *Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*
    - MGSE9-12.F.IF.5 *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*

17. RATIONAL EQUATIONS AND FUNCTIONS

- ### SOLVING RATIONAL EQUATIONS

    - MGSE9-12.F.IF.7d *Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.*
    - MGSE9-12.A.REI.11 *Using graphs, tables, or successive approximations, show that the solution to the equation f(x) = g(x) is the x-value where the y-values of f(x) and g(x) are the same.*
    - MGSE9-12.A.REI.2 *Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.*

- ### MODELING SITUATIONS WITH RATIONAL FUNCTIONS

    - MGSE9-12.A.SSE.1a *Interpret parts of an expression, such as terms, factors, and coefficients, in context.*

- MGSE9-12.A.SSE.1b *Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.*
- MGSE9-12.A.REI.2 *Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.*
- MGSE9-12.A.CED.2 *Create linear, quadratic, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
- MGSE9-12.A.CED.3 *Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.*

18. NONLINEAR FUNCTIONS

- **LINEAR VERSUS NONLINEAR FUNCTIONS**
  - MGSE9-12.F.IF.6 *Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
  - MGSE9-12.F.IF.4 *Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*

- **ABSOLUTE VALUE FUNCTIONS**
  - MGSE9-12.F.IF.5 *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
  - MGSE9-12.F.IF.7b *Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.*

19. TRANSFORMING FUNCTIONS

- **PARENT FUNCTIONS**
  - MGSE9-12.F.BF.3 *Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
  - MGSE9-12.F.IF.7b *Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.*
  - MGSE9-12.F.IF.4 *Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*
  - MGSE9-12.F.IF.5 *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
  - MGSE9-12.F.IF.7e *Graph exponential and logarithmic functions, showing intercepts and end behavior.*
  - MGSE9-12.F.IF.7c *Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.*

- **TRANSFORMATIONS OF PARENT FUNCTIONS**
  - MGSE9-12.F.BF.3 *Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
  - MGSE9-12.F.IF.7e *Graph exponential and logarithmic functions, showing intercepts and end behavior.*
  - MGSE9-12.F.IF.7b *Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.*
  - MGSE9-12.F.IF.7c *Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.*
  - MGSE9-12.F.IF.5 *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
  - MGSE9-12.F.IF.4 *Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function*

is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.

- **MULTIPLE TRANSFORMATIONS OF PARENT FUNCTIONS**
  - **MGSE9-12.F.BF.3** *Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
  - **MGSE9-12.F.IF.7c** *Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.*
  - **MGSE9-12.F.IF.7b** *Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.*
  - **MGSE9-12.F.IF.5** *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
  - **MGSE9-12.F.IF.4** *Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*
  - **MGSE9-12.F.IF.7e** *Graph exponential and logarithmic functions, showing intercepts and end behavior.*

20. WORKING WITH FUNCTIONS

- **MULTIPLE REPRESENTATIONS OF FUNCTIONS**
  - **MGSE9-12.F.IF.9** *Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).*
  - **MGSE9-12.F.IF.4** *Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*
  - **MGSE9-12.A.CED.2** *Create linear, quadratic, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

- **ARITHMETIC OPERATIONS ON FUNCTIONS**
  - **MGSE9-12.F.BF.1b** *Combine standard function types using arithmetic operations in contextual situations (Adding, subtracting, and multiplying functions of different types).*

21. SYSTEMS OF EQUATIONS

- **SOLVING THREE-VARIABLE SYSTEMS OF LINEAR EQUATIONS**
  - **MGSE9-12.A.CED.2** *Create linear, quadratic, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
  - **MGSE9-12.A.CED.3** *Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.*

- **SYSTEMS OF NONLINEAR EQUATIONS**
  - **MGSE9-12.A.REI.11** *Using graphs, tables, or successive approximations, show that the solution to the equation f(x) = g(x) is the x-value where the y-values of f(x) and g(x) are the same.*

22. STATISTICAL DESIGN AND ANALYSIS

- **ANALYZING STATISTICAL SAMPLES**
  - **MGSE9-12.S.IC.1** *Understand statistics as a process for making inferences about population parameters based on a random sample from that population.*
  - **MGSE9-12.S.IC.2** *Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.*
  - **MGSE9-12.S.IC.4** *Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.*

- **EXPERIMENTAL AND OBSERVATIONAL DESIGN**
  - **MGSE9-12.S.IC.3** *Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.*

- **CONCLUSIONS IN DATA**
  - **GSE9-12.S.IC.5** *Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.*
  - **MGSE9-12.S.IC.6** *Evaluate reports based on data.*

23. STATISTICS

- **USING STATISTICAL MEASURES TO COMPARE DATA SETS**
  - **MGSE9-12.S.ID.2** *Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.*

- **NORMAL DISTRIBUTION**
  - **MGSE9-12.S.ID.4** *Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.*
  - **MGSE9-12.S.IC.4** *Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.*