

Ohio Tutorials are designed specifically for the Ohio Learning Standards to prepare students for the Ohio State Tests and end-of-course exams.

Math Tutorials offer targeted instruction, practice and review designed to develop computational fluency, deepen conceptual understanding, and apply mathematical practices. They automatically identify and address learning gaps down to elementary-level content, using adaptive remediation to bring students to grade-level no matter where they start. Students engage with the content in an interactive, feedback-rich environment as they progress through standards-aligned modules. By constantly honing the ability to apply their knowledge in abstract and real world scenarios, students build the depth of knowledge and higher order skills required to demonstrate their mastery when put to the test.

In each module, the Learn It and Try It make complex ideas accessible through focused content, guided analysis, multi-modal representations, and personalized feedback as students reason through increasingly challenging problems. The Review It offers a high impact summary of key concepts and relates those concepts to students' lives. The Test It assesses students' mastery of the module's concepts, providing granular performance data to students and teachers after each attempt. To help students concentrate on the content most relevant to them, unit-level pretests and posttests can quickly identify where students are strong and where they're still learning.

## 1. REAL NUMBER SYSTEM

### • LAWS OF EXPONENTS

- **OH.Math.HSA.SSE.2** Use the structure of an expression to identify ways to rewrite it.
- **OH.Math.HSA.APR.6** Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.
- **OH.Math.HSA.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- **OH.Math.HSN.RN.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
- **OH.Math.HSN.RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.

### • OPERATIONS ON RATIONAL AND IRRATIONAL NUMBERS

- **OH.Math.HSN.RN.3** Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

### • MONITORING PRECISION AND ACCURACY

- **OH.Math.HSN.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
- **OH.Math.HSN.Q.2** Define appropriate quantities for the purpose of descriptive modeling.
- **OH.Math.HSN.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

## 2. EQUATIONS AND INEQUALITIES

### • ONE-STEP EQUATIONS AND INEQUALITIES

- **OH.Math.HSA.CED.1a** Focus on applying linear and simple exponential expressions.
- **OH.Math.HSA.REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- **OH.Math.HSA.CED.1b** Focus on applying simple quadratic expressions.

- **OH.Math.HSA.CED.3a** While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations.
- **OH.Math.HSA.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- **OH.Math.HSF.BF.1a.ii** Focus on situations that exhibit quadratic or exponential relationships.

#### ● **MULTI-STEP EQUATIONS AND INEQUALITIES**

- **OH.Math.HSA.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- **OH.Math.HSA.REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

#### ● **AXIOMS OF EQUALITY**

- **OH.Math.HSA.SSE.2** Use the structure of an expression to identify ways to rewrite it.
- **OH.Math.HSA.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

#### ● **LITERAL EQUATIONS**

- **OH.Math.HSA.REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- **OH.Math.HSA.CED.1a** Focus on applying linear and simple exponential expressions.
- **OH.Math.HSA.CED.4b** Focus on formulas in which the variable of interest is linear.
- **OH.Math.HSA.CED.4c** Focus on formulas in which the variable of interest is linear or square.
- **OH.Math.HSA.CED.4d** While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations.
- **OH.Math.HSA.CED.4a** Focus on formulas in which the variable of interest is linear or square.

### 3. WRITING EXPRESSIONS AND EQUATIONS

#### ● **FORMULATING AND SIMPLIFYING ALGEBRAIC EXPRESSIONS**

- **OH.Math.HSF.BF.1a.ii** Focus on situations that exhibit quadratic or exponential relationships.
- **OH.Math.HSA.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.
- **OH.Math.HSA.SSE.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **OH.Math.HSA.SSE.2** Use the structure of an expression to identify ways to rewrite it.

#### ● **FORMULATING AND SOLVING EQUATIONS FROM WORD PROBLEMS**

- **OH.Math.HSF.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- **OH.Math.HSF.LE.1b** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- **OH.Math.HSF.BF.1a.ii** Focus on situations that exhibit quadratic or exponential relationships.
- **OH.Math.HSA.CED.2a** Focus on applying linear and simple exponential expressions.
- **OH.Math.HSF.IF.4a** Focus on linear and exponential functions.
- **OH.Math.HSF.IF.4b** Focus on linear, quadratic, and exponential functions.
- **OH.Math.HSF.BF.1a.i** Focus on linear and exponential functions.
- **OH.Math.HSA.CED.1c** Extend to include more complicated function situations with the option to solve with technology.
- **OH.Math.HSA.CED.3a** While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations.
- **OH.Math.HSA.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.

- **FORMULATING AND SOLVING INEQUALITIES FROM WORD PROBLEMS**

- **OH.Math.HSA.REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- **OH.Math.HSA.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.

## 4. FUNCTIONS

- **FUNCTIONS AND RELATIONS**

- **OH.Math.HSF.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- **OH.Math.HSF.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **OH.Math.HSF.BF.4a** Informally determine the input of a function when the output is known.
- **OH.Math.HSF.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- **OH.Math.HSF.IF.7c** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

- **DOMAIN AND RANGE**

- **OH.Math.HSF.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **OH.Math.HSF.IF.5c** Emphasize the selection of a type of function for a model based on behavior of data and context.

- **EVALUATING FUNCTIONS**

- **OH.Math.HSF.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **OH.Math.HSF.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- **OH.Math.HSF.BF.4a** Informally determine the input of a function when the output is known.

## 5. GRAPHS OF LINEAR EQUATIONS AND INEQUALITIES

- **SLOPE**

- **OH.Math.HSF.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- **OH.Math.HSF.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **OH.Math.HSF.BF.4a** Informally determine the input of a function when the output is known.
- **OH.Math.HSG.GPE.5** Justify the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems, e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point.

- **GRAPHING AND ANALYZING LINEAR FUNCTIONS**

- **OH.Math.HSF.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **OH.Math.HSF.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- **OH.Math.HSF.BF.4a** Informally determine the input of a function when the output is known.
- **OH.Math.HSF.IF.7a** Graph linear functions and indicate intercepts.
- **OH.Math.HSF.IF.4a** Focus on linear and exponential functions.

- **OH.Math.HSF.IF.4b** Focus on linear, quadratic, and exponential functions.

- **GRAPHING AND MANIPULATING  $Y = MX + B$**

- **OH.Math.HSF.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **OH.Math.HSF.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- **OH.Math.HSF.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- **OH.Math.HSF.IF.7a** Graph linear functions and indicate intercepts.
- **OH.Math.HSS.ID.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- **OH.Math.HSF.LE.5** Interpret the parameters in a linear or exponential function in terms of a context.
- **OH.Math.HSF.LE.1b** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

- **GRAPHS OF LINEAR INEQUALITIES**

- **OH.Math.HSA.REI.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
- **OH.Math.HSA.REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- **OH.Math.HSA.CED.3a** While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations.

## 6. LINEAR EQUATIONS

- **SLOPE-INTERCEPT FORM OF A LINEAR EQUATION**

- **OH.Math.HSS.ID.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- **OH.Math.HSF.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **OH.Math.HSF.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- **OH.Math.HSF.IF.7a** Graph linear functions and indicate intercepts.
- **OH.Math.HSA.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- **OH.Math.HSG.GPE.5** Justify the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems, e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point.
- **OH.Math.HSF.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

- **POINT-SLOPE FORM OF A LINEAR EQUATION**

- **OH.Math.HSA.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- **OH.Math.HSF.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **OH.Math.HSF.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- **OH.Math.HSF.IF.7a** Graph linear functions and indicate intercepts.
- **OH.Math.HSG.GPE.5** Justify the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems, e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point.

## 7. TWO-VARIABLE LINEAR SYSTEMS

### ● SOLVING SYSTEMS OF LINEAR EQUATIONS: GUESS AND CHECK

- **OH.Math.HSF.BF.1a.ii** Focus on situations that exhibit quadratic or exponential relationships.
- **OH.Math.HSA.REI.6a** Limit to pairs of linear equations in two variables.

### ● SOLVING SYSTEMS OF LINEAR EQUATIONS: GRAPHING

- **OH.Math.HSA.CED.3a** While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations.
- **OH.Math.HSA.REI.11** Explain why the  $x$ -coordinates of the points where the graphs of the equation  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations.
- **OH.Math.HSA.REI.6a** Limit to pairs of linear equations in two variables.

### ● SOLVING SYSTEMS OF LINEAR EQUATIONS: SUBSTITUTION

- **OH.Math.HSA.REI.5** Verify that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

### ● SOLVING SYSTEMS OF LINEAR EQUATIONS: ELIMINATION

- **OH.Math.HSA.REI.5** Verify that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
- **OH.Math.HSA.REI.6a** Limit to pairs of linear equations in two variables.

## 8. LINEAR SYSTEMS

### ● SOLVING SYSTEMS OF LINEAR INEQUALITIES

- **OH.Math.HSA.CED.3a** While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations.
- **OH.Math.HSA.REI.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
- **OH.Math.HSA.CED.2a** Focus on applying linear and simple exponential expressions.

### ● SOLVING THREE-VARIABLE SYSTEMS OF LINEAR EQUATIONS

- **OH.Math.HSA.REI.5** Verify that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

## 9. EXPONENTIAL FUNCTIONS, EQUATIONS, AND INEQUALITIES

### ● EXPONENTIAL FUNCTIONS

- **OH.Math.HSA.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.
- **OH.Math.HSA.SSE.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **OH.Math.HSA.SSE.3c** Use the properties of exponents to transform expressions for exponential functions.
- **OH.Math.HSF.IF.7e** Graph simple exponential functions, indicating intercepts and end behavior.
- **OH.Math.HSF.LE.1a** Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
- **OH.Math.HSF.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- **OH.Math.HSF.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- **OH.Math.HSF.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.

- **OH.Math.HSF.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **OH.Math.HSF.BF.4a** Informally determine the input of a function when the output is known.
- **OH.Math.HSA.REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- **OH.Math.HSF.LE.5** Interpret the parameters in a linear or exponential function in terms of a context.
- **OH.Math.HSF.IF.7f** Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- **OH.Math.HSF.IF.7g** Graph rational functions, identifying zeros and asymptotes when factoring is reasonable, and indicating end behavior.
- **OH.Math.HSF.LE.1c** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- **OH.Math.HSA.CED.1a** Focus on applying linear and simple exponential expressions.
- **OH.Math.HSA.CED.1c** Extend to include more complicated function situations with the option to solve with technology.
- **OH.Math.HSA.CED.2a** Focus on applying linear and simple exponential expressions.
- **OH.Math.HSA.CED.2c** Extend to include more complicated function situations with the option to graph with technology.
- **OH.Math.HSF.BF.1a.i** Focus on linear and exponential functions.
- **OH.Math.HSF.BF.1a.ii** Focus on situations that exhibit quadratic or exponential relationships.

### ● EXPONENTIAL GROWTH AND DECAY

- **OH.Math.HSA.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.
- **OH.Math.HSA.SSE.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **OH.Math.HSF.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- **OH.Math.HSF.LE.5** Interpret the parameters in a linear or exponential function in terms of a context.
- **OH.Math.HSF.LE.1a** Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
- **OH.Math.HSF.LE.1c** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- **OH.Math.HSA.CED.2a** Focus on applying linear and simple exponential expressions.
- **OH.Math.HSA.SSE.3c** Use the properties of exponents to transform expressions for exponential functions.
- **OH.Math.HSF.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.
- **OH.Math.HSF.LE.1b** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- **OH.Math.HSA.CED.2c** Extend to include more complicated function situations with the option to graph with technology.
- **OH.Math.HSF.IF.9a** Focus on linear and exponential functions.
- **OH.Math.HSF.IF.9b** Focus on linear, quadratic, and exponential functions.
- **OH.Math.HSF.BF.1a.i** Focus on linear and exponential functions.
- **OH.Math.HSF.BF.1a.ii** Focus on situations that exhibit quadratic or exponential relationships.

### ● SOLVING EXPONENTIAL INEQUALITIES

- **OH.Math.HSF.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- **OH.Math.HSA.CED.2a** Focus on applying linear and simple exponential expressions.
- **OH.Math.HSA.CED.2c** Extend to include more complicated function situations with the option to graph with technology.
- **OH.Math.HSA.SSE.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **OH.Math.HSF.LE.1c** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

## 10. SEQUENCES

### ● SEQUENCES

- **OH.Math.HSF.IF.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
- **OH.Math.HSF.BF.2** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
- **OH.Math.HSF.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

- **ARITHMETIC AND GEOMETRIC SEQUENCES**

- **OH.Math.HSF.BF.2** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
- **OH.Math.HSF.IF.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
- **OH.Math.HSF.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- **OH.Math.HSF.BF.1a.i** Focus on linear and exponential functions.
- **OH.Math.HSF.BF.1a.ii** Focus on situations that exhibit quadratic or exponential relationships.

## 11. POLYNOMIALS

- **POLYNOMIAL BASICS**

- **OH.Math.HSA.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.
- **OH.Math.HSA.SSE.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **OH.Math.HSA.SSE.2** Use the structure of an expression to identify ways to rewrite it.
- **OH.Math.HSA.APR.1b** Extend to polynomial expressions beyond those expressions that simplify to forms that are linear or quadratic.

- **ADDITION AND SUBTRACTION OF POLYNOMIALS**

- **OH.Math.HSA.APR.1b** Extend to polynomial expressions beyond those expressions that simplify to forms that are linear or quadratic.
- **OH.Math.HSA.SSE.2** Use the structure of an expression to identify ways to rewrite it.

- **MULTIPLICATION OF POLYNOMIALS**

- **OH.Math.HSA.APR.1b** Extend to polynomial expressions beyond those expressions that simplify to forms that are linear or quadratic.
- **OH.Math.HSA.SSE.2** Use the structure of an expression to identify ways to rewrite it.

## 12. FACTORING

- **FACTORING QUADRATIC TRINOMIALS**

- **OH.Math.HSA.SSE.3a** Factor a quadratic expression to reveal the zeros of the function it defines.
- **OH.Math.HSA.REI.4b** Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for  $x^2 = 49$ ; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.
- **OH.Math.HSA.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.
- **OH.Math.HSA.SSE.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity.

- **FACTORING SPECIAL CASES**

- **OH.Math.HSA.SSE.2** Use the structure of an expression to identify ways to rewrite it.
- **OH.Math.HSA.APR.4** Prove polynomial identities and use them to describe numerical relationships.
- **OH.Math.HSA.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.
- **OH.Math.HSA.SSE.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **OH.Math.HSA.APR.3** Identify zeros of polynomials, when factoring is reasonable, and use the zeros to construct a rough graph of the function defined by the polynomial.

- **FACTORING HIGHER-ORDER POLYNOMIALS**

- **OH.Math.HSA.APR.3** Identify zeros of polynomials, when factoring is reasonable, and use the zeros to construct a rough graph of the function defined by the polynomial.
- **OH.Math.HSA.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.
- **OH.Math.HSA.SSE.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **OH.Math.HSA.SSE.2** Use the structure of an expression to identify ways to rewrite it.
- **OH.Math.HSA.APR.4** Prove polynomial identities and use them to describe numerical relationships.

## 13. GRAPHS OF QUADRATIC FUNCTIONS

- **QUADRATIC FUNCTIONS**

- **OH.Math.HSF.IF.4b** Focus on linear, quadratic, and exponential functions.
- **OH.Math.HSA.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.
- **OH.Math.HSA.SSE.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **OH.Math.HSA.SSE.3b** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- **OH.Math.HSF.BF.1a.ii** Focus on situations that exhibit quadratic or exponential relationships.

- **ANALYZING GRAPHS OF QUADRATIC FUNCTIONS**

- **OH.Math.HSA.SSE.2** Use the structure of an expression to identify ways to rewrite it.
- **OH.Math.HSF.IF.4b** Focus on linear, quadratic, and exponential functions.
- **OH.Math.HSF.IF.9b** Focus on linear, quadratic, and exponential functions.
- **OH.Math.HSF.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **OH.Math.HSF.IF.7b** Graph quadratic functions and indicate intercepts, maxima, and minima.
- **OH.Math.HSA.REI.4b** Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for  $x^2 = 49$ ; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.
- **OH.Math.HSF.IF.5b** Focus on linear, quadratic, and exponential functions.
- **OH.Math.HSA.APR.3** Identify zeros of polynomials, when factoring is reasonable, and use the zeros to construct a rough graph of the function defined by the polynomial.

- **REPRESENTATIONS OF QUADRATIC FUNCTIONS**

- **OH.Math.HSA.SSE.2** Use the structure of an expression to identify ways to rewrite it.
- **OH.Math.HSA.REI.4c** Derive the quadratic formula using the method of completing the square.
- **OH.Math.HSF.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **OH.Math.HSA.CED.2c** Extend to include more complicated function situations with the option to graph with technology.
- **OH.Math.HSF.IF.4b** Focus on linear, quadratic, and exponential functions.
- **OH.Math.HSF.IF.8a.i** Focus on completing the square to quadratic functions with the leading coefficient of 1.
- **OH.Math.HSF.IF.9b** Focus on linear, quadratic, and exponential functions.
- **OH.Math.HSF.BF.1a.ii** Focus on situations that exhibit quadratic or exponential relationships.
- **OH.Math.HSA.CED.2b** Focus on applying simple quadratic expressions.
- **OH.Math.HSA.SSE.3a** Factor a quadratic expression to reveal the zeros of the function it defines.
- **OH.Math.HSA.REI.4a** Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions.
- **OH.Math.HSF.BF.3a** Focus on transformations of graphs of quadratic functions, except for  $f(kx)$ .

## 14. SOLVING QUADRATIC FUNCTIONS

## • SOLVING QUADRATIC EQUATIONS BY FACTORING

- **OH.Math.HSA.SSE.3a** Factor a quadratic expression to reveal the zeros of the function it defines.
- **OH.Math.HSA.REI.4b** Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for  $x^2 = 49$ ; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.
- **OH.Math.HSA.APR.3** Identify zeros of polynomials, when factoring is reasonable, and use the zeros to construct a rough graph of the function defined by the polynomial.
- **OH.Math.HSA.APR.4** Prove polynomial identities and use them to describe numerical relationships.
- **OH.Math.HSF.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **OH.Math.HSF.IF.7b** Graph quadratic functions and indicate intercepts, maxima, and minima.
- **OH.Math.HSA.CED.2b** Focus on applying simple quadratic expressions.
- **OH.Math.HSA.CED.2c** Extend to include more complicated function situations with the option to graph with technology.
- **OH.Math.HSF.IF.9b** Focus on linear, quadratic, and exponential functions.
- **OH.Math.HSF.BF.1a.ii** Focus on situations that exhibit quadratic or exponential relationships.

## • COMPLETING THE SQUARE

- **OH.Math.HSA.SSE.3b** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- **OH.Math.HSA.REI.4a** Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions.
- **OH.Math.HSA.REI.4b** Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for  $x^2 = 49$ ; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.
- **OH.Math.HSA.SSE.2** Use the structure of an expression to identify ways to rewrite it.
- **OH.Math.HSA.REI.4c** Derive the quadratic formula using the method of completing the square.
- **OH.Math.HSF.IF.7b** Graph quadratic functions and indicate intercepts, maxima, and minima.

## • QUADRATIC FORMULA

- **OH.Math.HSA.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.
- **OH.Math.HSA.SSE.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **OH.Math.HSA.REI.4b** Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for  $x^2 = 49$ ; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.
- **OH.Math.HSA.REI.4c** Derive the quadratic formula using the method of completing the square.
- **OH.Math.HSF.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **OH.Math.HSA.REI.4a** Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions.
- **OH.Math.HSF.BF.1a.ii** Focus on situations that exhibit quadratic or exponential relationships.

## 15. PARENT FUNCTIONS AND TRANSFORMATIONS

### • LINEAR AND EXPONENTIAL PARENT FUNCTIONS

- **OH.Math.HSA.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- **OH.Math.HSF.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- **OH.Math.HSF.IF.4a** Focus on linear and exponential functions.
- **OH.Math.HSF.IF.4b** Focus on linear, quadratic, and exponential functions.
- **OH.Math.HSF.IF.5b** Focus on linear, quadratic, and exponential functions.
- **OH.Math.HSF.IF.9a** Focus on linear and exponential functions.

- **OH.Math.HSF.IF.9b** Focus on linear, quadratic, and exponential functions.
- **OH.Math.HSF.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **OH.Math.HSF.LE.1c** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

#### ● QUADRATIC PARENT FUNCTION

- **OH.Math.HSA.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- **OH.Math.HSF.IF.7b** Graph quadratic functions and indicate intercepts, maxima, and minima.
- **OH.Math.HSF.IF.5b** Focus on linear, quadratic, and exponential functions.
- **OH.Math.HSF.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

#### ● TRANSFORMATIONS OF THE LINEAR AND EXPONENTIAL PARENT FUNCTIONS

- **OH.Math.HSG.CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not, e.g., translation versus horizontal stretch.
- **OH.Math.HSG.CO.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- **OH.Math.HSF.BF.3a** Focus on transformations of graphs of quadratic functions, except for  $f(kx)$ .

#### ● TRANSFORMATIONS OF THE QUADRATIC PARENT FUNCTION

- **OH.Math.HSG.CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not, e.g., translation versus horizontal stretch.
- **OH.Math.HSF.BF.3a** Focus on transformations of graphs of quadratic functions, except for  $f(kx)$ .
- **OH.Math.HSA.CED.2b** Focus on applying simple quadratic expressions.
- **OH.Math.HSF.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

## 16. NONLINEAR FUNCTIONS

#### ● LINEAR VERSUS NONLINEAR FUNCTIONS

- **OH.Math.HSF.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- **OH.Math.HSF.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- **OH.Math.HSF.BF.1b** Combine standard function types using arithmetic operations.
- **OH.Math.HSF.BF.1c** Compose functions.
- **OH.Math.HSF.LE.1a** Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
- **OH.Math.HSF.LE.1b** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- **OH.Math.HSF.LE.1c** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- **OH.Math.HSF.IF.4a** Focus on linear and exponential functions.
- **OH.Math.HSF.IF.4b** Focus on linear, quadratic, and exponential functions.
- **OH.Math.HSF.IF.9b** Focus on linear, quadratic, and exponential functions.

#### ● ABSOLUTE VALUE FUNCTIONS

- **OH.Math.HSF.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **OH.Math.HSF.BF.3a** Focus on transformations of graphs of quadratic functions, except for  $f(kx)$ .
- **OH.Math.HSF.IF.7c** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- **OH.Math.HSF.IF.4b** Focus on linear, quadratic, and exponential functions.

## ● SYSTEMS OF NONLINEAR EQUATIONS

- **OH.Math.HSA.REI.7** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
- **OH.Math.HSA.REI.5** Verify that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
- **OH.Math.HSA.REI.11** Explain why the  $x$ -coordinates of the points where the graphs of the equation  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations.
- **OH.Math.HSA.CED.3a** While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations.
- **OH.Math.HSF.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

## 17. WORKING WITH FUNCTIONS

### ● ARITHMETIC OPERATIONS ON FUNCTIONS

- **OH.Math.HSF.BF.1b** Combine standard function types using arithmetic operations.

### ● MULTIPLE REPRESENTATIONS OF FUNCTIONS

- **OH.Math.HSF.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- **OH.Math.HSA.CED.2c** Extend to include more complicated function situations with the option to graph with technology.
- **OH.Math.HSF.IF.4a** Focus on linear and exponential functions.
- **OH.Math.HSF.IF.4b** Focus on linear, quadratic, and exponential functions.
- **OH.Math.HSF.IF.9a** Focus on linear and exponential functions.
- **OH.Math.HSF.IF.9b** Focus on linear, quadratic, and exponential functions.
- **OH.Math.HSA.CED.2b** Focus on applying simple quadratic expressions.
- **OH.Math.HSF.LE.1a** Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.

### ● INVERSE FUNCTIONS

- **OH.Math.HSF.BF.4b** Read values of an inverse function from a graph or a table, given that the function has an inverse.
- **OH.Math.HSF.BF.4c** Verify by composition that one function is the inverse of another.
- **OH.Math.HSF.BF.4d** Find the inverse of a function algebraically, given that the function has an inverse.
- **OH.Math.HSF.BF.4a** Informally determine the input of a function when the output is known.
- **OH.Math.HSF.BF.4e** Produce an invertible function from a non-invertible function by restricting the domain.

## 18. STATISTICS

### ● DATA ANALYSIS

- **OH.Math.HSS.ID.1** Represent data with plots on the real number line (dot plots, histograms, and box plots) in the context of real-world applications using the GAISE model.
- **OH.Math.HSS.ID.2** In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare center (median and mean) and spread (mean absolute deviation, interquartile range, and standard deviation) of two or more different data sets.

- **OH.Math.HSS.ID.3** *In the context of real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).*

- **FREQUENCY TABLES**

- **OH.Math.HSS.ID.5** *Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.*
- **OH.Math.HSS.CP.4** *Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.*

- **SCATTERPLOTS**

- **OH.Math.HSS.ID.6b** *Informally assess the fit of a function by discussing residuals.*
- **OH.Math.HSS.ID.6c** *Fit a linear function for a scatterplot that suggests a linear association.*
- **OH.Math.HSF.IF.6** *Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
- **OH.Math.HSS.ID.7** *Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.*

- **SCATTERPLOTS AND MODELING**

- **OH.Math.HSS.ID.6a** *Fit a function to the data; use functions fitted to data to solve problems in the context of the data.*
- **OH.Math.HSS.ID.6b** *Informally assess the fit of a function by discussing residuals.*
- **OH.Math.HSS.ID.8** *Compute (using technology) and interpret the correlation coefficient of a linear fit.*
- **OH.Math.HSS.ID.6c** *Fit a linear function for a scatterplot that suggests a linear association.*
- **OH.Math.HSS.ID.7** *Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.*
- **OH.Math.HSF.LE.1a** *Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.*
- **OH.Math.HSF.LE.1c** *Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.*